Adaptivno upravljanje s referentnim modelom i parametarskom adaptacijom

Adaptivno i robusno upravljanje

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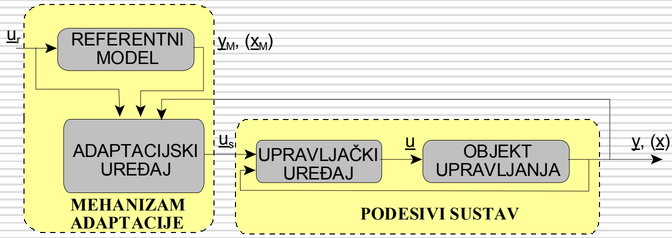
Sadržaj predavanja

- Izvod algoritma adaptivnog upravljanja s referentnim modelom i parametarskom adaptacijom
- Dokaz stabilnosti
  - Kriterij stabilnosti Ljapunova
  - Kriterij stabilnosti Popova
- Normiranje signala algoritma
- Primjer

Parametarska adaptacija

struktura algoritma



$$\dot{\underline{x}}_M = \underline{A}_M \underline{x}_M + \underline{b}_M u_r,$$
$$y_m = \underline{c}_M \underline{x}_M,$$

$$\dot{\underline{x}} = \underline{A}\underline{x} + \underline{b} u_s,$$
$$y = \underline{c}\underline{x},$$

$$\underline{e} = \underline{x}_M - \underline{x},$$

Struktura algoritma

- Diferencijalna jednadžba pogreške:

$$\dot{\mathbf{e}} = \mathbf{A_M e} + \mu_1$$
$$\mu_1 = (\mathbf{A_M} - \mathbf{A})\,\mathbf{x} + \mathbf{b_M} u_r - \mathbf{b} u_s$$

- Zahtjev na sustav da bi pogreška težila k nuli:

$$\mu_1 = \mathbf{0}$$

Parametarska adaptacija

struktura algoritma

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Uz izbor varijabli stanja

$$\underline{A}_M = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_{M0} & -a_{M1} & -a_{M2} & \cdots & -a_{Mn-1} \end{bmatrix}, \quad \begin{aligned} \underline{b}_M^T &= [0\ 0\ 0\ \cdots\ b_{M0}], \\ \underline{c}_M &= [c_{M1}\ c_{M2}\ c_{M3}\ \cdots\ c_{Mn}], \end{aligned}$$

$$\underline{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix}, \quad \begin{aligned} \underline{b}^T &= [0\ 0\ 0\ \cdots\ b_0], \\ \underline{c} &= [c_1\ c_2\ c_3\ \cdots\ c_n]. \end{aligned}$$

Oblik algoritma upravljanja

$$u_s = \boxed{\frac{1}{b_0}} \underline{\tilde{a}}^T \underline{x} + \boxed{\frac{b_{M0}}{b_0}} u_r,$$
$$\underline{\tilde{a}}^T = -\left[a_{M0} - a_0 \quad a_{M1} - a_1 \quad a_{M2} - a_2 \quad \cdots \quad a_{Mn-1} - a_{n-1}\right]$$

$$u_s = k_d(t)\, u_r(t) + \underline{k}_p^T(t)\, \underline{x}(t)$$

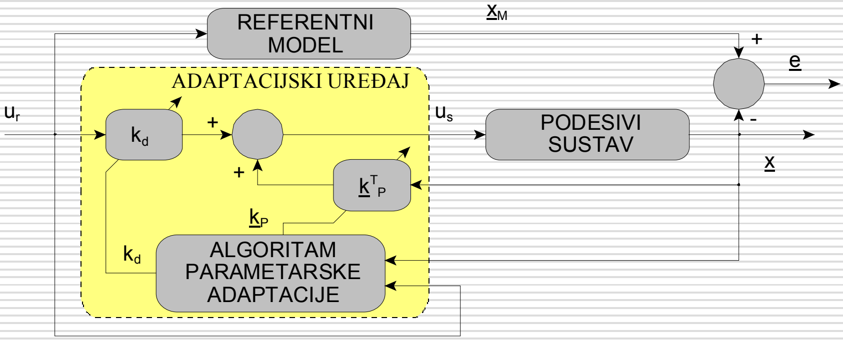
Parametarska adaptacija

struktura algoritma

$$u_s = k_d(t)\,u_r(t) + \underline{k}_p^T(t)\,\underline{x}(t)$$

$k_d(t)$ - vremenski promjenjivi parametar adaptacije u direktnoj grani,

$k_p^T(t)$ - vektor vremenski promjenjivih parametara adaptacije u povratnoj vezi



Parametarska adaptacija

opći oblik varijabli stanja

- Nemogućnost prikaza algoritma algebarskom jednadžbom ako
  - sustav nije moguće prikazati u kanoničkoj osmotrivoj formi
    - (matrica A ima samo elemente u zadnjem retku i jedinice u pomoćnoj dijagonali)
    - nemogućnost mjerenja traženih varijabli
  - matrica C ima promjenjive elemente
- Potreban prikaz sustava da izlazna varijabla bude jedna od varijabli stanja
  - jednadžba $\underline{\mu}_1 = \underline{0}$ ne mora imati rješenje
  - traži se minimum norme vektora $\underline{\mu}_1$
  - težinska norma (*euklidska norma)*

$$f_n^2(\underline{\mu}_1) = \underline{\mu}_1^T \cdot \underline{W} \cdot \underline{\mu}_1,$$

- W- pozitivno definitna matrica težinskih koeficijenata
- f_n – norma vektora

Parametarska adaptacija
opći oblik varijabli stanja

$$f_n^2(\underline{\mu}_1) = \underline{\mu}_1^T \cdot \underline{W} \cdot \underline{\mu}_1,$$

Kvadrat norme vektora μ_1

$$f_n^2(\underline{\mu}_1) = \underline{b}^T \underline{W} \underline{b} u_s^2 - 2(\underline{b}^T \underline{W} \tilde{\underline{A}} \underline{x} + \underline{b}_M^T \underline{W} \underline{b} u_r) u_s + (\tilde{\underline{A}} \underline{x} + \underline{b}_M u_r)^T \underline{W} (\tilde{\underline{A}} \underline{x} + \underline{b}_M u_r),$$

$$\tilde{\underline{A}} = \underline{A}_M - \underline{A}.$$

Minimum norme vektora μ_1 → minimum kvadrata norme vektora μ_1

$$\frac{\partial f_n^2(\underline{\mu}_1)}{\partial u_s} = 0.$$

$$\frac{\partial f_n^2(\underline{\mu}_1)}{\partial u_s} = 2\left[\underline{b}^T \underline{W}\left(\underline{b} - \tilde{\underline{A}} \frac{\partial \underline{x}}{\partial u_s}\right) u_s + \left(\frac{\partial \underline{x}^T}{\partial u_s} \tilde{\underline{A}}^T - \underline{b}^T\right) \underline{W} \left\{\tilde{\underline{A}} \underline{x} + \underline{b}_M u_r\right\}\right].$$

Parametarska adaptacija
opći oblik varijabli stanja

Izražavanje vektora varijabli stanja u vremenskoj domeni

$$\dot{\underline{x}} = \underline{A} \underline{x} + \underline{b} u_s, \text{ (uz } \underline{x}(0)=\underline{0})$$

$$\underline{x}(t) = \int_0^t \underline{\Phi}(t-\tau) \underline{b} u_s(\tau) d\tau,$$

$$\underline{\Phi}(t) = \mathcal{L}^{-1}\left[(s\underline{I} - \underline{A})^{-1}\right],$$

I - jedinična matrica,
s - Laplaceov operator.

$$\frac{\partial \underline{x}}{\partial u_s} = \int_0^t \underline{\Phi}(t-\tau) \underline{b} d\tau = \underline{f}_x(t).$$

$$\frac{\partial f_n^2(\underline{\mu}_1)}{\partial u_s} = 0.$$

$$\frac{\partial f_n^2(\underline{\mu}_1)}{\partial u_s} = 2\left[\underline{b}^T \underline{W}\left(\underline{b} - \tilde{\underline{A}} \frac{\partial \underline{x}}{\partial u_s}\right) u_s + \left(\frac{\partial \underline{x}^T}{\partial u_s} \tilde{\underline{A}}^T - \underline{b}^T\right) \underline{W} \left\{\tilde{\underline{A}} \underline{x} + \underline{b}_M u_r\right\}\right].$$

$$u_s(t) = \left\{\underline{b}^T \underline{W}\left[\underline{b} - \tilde{\underline{A}} \underline{f}_x(t)\right]\right\}^{-1} \cdot \left[\underline{b}^T - \underline{f}_x^T(t) \cdot \tilde{\underline{A}}^T\right] \cdot \underline{W} \cdot (\tilde{\underline{A}} \underline{x} + \underline{b}_M u_r).$$

Parametarska adaptacija

opći oblik varijabli stanja

Izražavanje vektora varijabli stanja u vremenskoj domeni

$$\dot{\underline{x}} = \underline{A}\underline{x} + \underline{b}u_s, \text{ (uz } \underline{x}(0)=\underline{0})$$

$$\frac{\partial f_n^2(\underline{\mu}_1)}{\partial u_s} = 0.$$

$$\underline{\tilde{A}} = \underline{A}_M - \underline{A}.$$

$$\frac{\partial f_n^2(\underline{\mu}_1)}{\partial u_s} = 2\left[\underline{b}^T\underline{W}\left(\underline{b} - \underline{\tilde{A}}\frac{\partial \underline{x}}{\partial u_s}\right)u_s + \left(\frac{\partial \underline{x}^T}{\partial u_s}\underline{\tilde{A}}^T - \underline{b}^T\right)\underline{W}\left(\underline{\tilde{A}}\underline{x} + \underline{b}_M u_r\right)\right].$$

$$u_s(t) = \left\{\underline{b}^T\underline{W}\left[\underline{b} - \underline{\tilde{A}}\underline{f}_x(t)\right]\right\}^{-1}\cdot\left[\underline{b}^T - \underline{f}_x^T(t)\cdot\underline{\tilde{A}}^T\right]\cdot\underline{W}\cdot(\underline{\tilde{A}}\underline{x} + \underline{b}_M u_r).$$

Primjenom težinske matrice oblika W=[wij] sa svojstvom $w_{ij}=0$ i=1, 2, ... n-1, j=1, 2, ..., n-1, uz $w_{nn}=1$,

$$\boxed{u_s = k_d(t)\,u_r(t) + \underline{k}_p^T(t)\,\underline{x}(t)}$$

$$k_d(t) = \left\{\underline{b}^T\underline{W}\left[\underline{b} - \underline{\tilde{A}}\underline{f}_x(t)\right]\right\}^{-1}\cdot\left[\underline{b}^T - \underline{f}_x^T(t)\cdot\underline{\tilde{A}}^T\right]\cdot\underline{W}\cdot\underline{b}_M$$

$$\underline{k}_p^T(t) = \left\{\underline{b}^T\underline{W}\left[\underline{b} - \underline{\tilde{A}}\underline{f}_x(t)\right]\right\}^{-1}\cdot\left[\underline{b}^T - \underline{f}_x^T(t)\cdot\underline{\tilde{A}}^T\right]\cdot\underline{W}\cdot\underline{\tilde{A}}$$

Kriterij stabilnosti Ljapunova

$V(\underline{e}) > 0 \quad \forall\ \underline{e} \neq \underline{0}$ *(pozitivna definitnost),*

$\dot{V}(\underline{e}) < 0 \quad \forall\ \underline{e} \neq \underline{0}$ *(negativna definitnost),*

$V(\underline{e}) \to \infty \quad \forall\ \|\underline{e}\| \to \infty,$

$V(\underline{0}) = 0,$

$\underline{e}$ - argument funkcije Ljapunova,
V - funkcija Ljapunova,
$\dot{V}$ - derivacija funkcije Ljapunova.

$$V = \underline{e}^T P \underline{e} + \underline{\Phi}^T \Gamma^{-1} \underline{\Phi},$$

$$\dot{V} = -\underline{e}^T Q\, \underline{e} + f(\underline{\Phi})$$

$$-Q = A_M^T P + P A_M$$

$\underline{P}$ $\underline{\Gamma}$ - pozitivno definitne matrice
$\underline{Q}$ - pozitivno defiinitna matrica
$\underline{e}$ - vektor pogreške sustava prema referentnom modelu
$\underline{\Phi}$ - vektor pogreške parametara

Derivacija funkcije Ljapunova negativno definitna ako je $f(\underline{\Phi})=0$
$f(\underline{\Phi})=0$ ====> određenje parametara adaptacije k_d i $\underline{k}_p$

Kriterij stabilnosti Ljapunova

Uz sustav oblika

$$\dot{\underline{e}} = \underline{A}_M \underline{e} + \underline{\mu}_1,$$
$$\underline{\mu}_1 = (\underline{A}_M - \underline{A} - \underline{b}\,\underline{k}_p^T(t))\underline{x} + (\underline{b}_M - \underline{b}\,k_d(t))u_r$$

Pogreška parametara

$$(\underline{b}_M - \underline{b}\,k_d(t)) \qquad (\underline{A}_M - \underline{A} - \underline{b}\,\underline{k}_p^T(t))$$

$$V = \underline{e}^T P \underline{e} + \underline{\Phi}^T \Gamma^{-1} \underline{\Phi},$$

$$\psi_d = b_{M0} - b_0 k_d(t),$$
$$\underline{\psi}_p^T = [\,-(a_{M0} - a_0) - b_0 k_{p1}(t) \quad -(a_{M1} - a_1) - b_0 k_{p2}(t) \quad \dots \quad -(a_{mn-1} - a_{n-1}) - b_0 k_{pn}(t)],$$

$$V = \underline{e}^T \underline{P}\,\underline{e} + \psi_d \gamma_d^{-1} \psi_d + \underline{\psi}_p^T \underline{\Gamma}_p^{-1} \underline{\psi}_p,$$

P	-	pozitivno definitna simetrična matrica dimenzije *nxn*,
Γ_p^{-1}	-	pozitivno definitna dijagonalna matrica dimenzije *nxn*,
γ_d^{-1}	-	pozitivna konstanta,
$-[a_0\ a_1\ \dots\ a_{n-1}]$	-	posljednji redak matrice stanja sustava prema jednadžbi (3–5),
$-[a_{M0}\ a_{M1}\ \dots\ a_{Mn-1}]$	-	posljednji redak matrice stanja referentnog modela prema jednadžbi (3–5),
b_0, b_{m0}	-	elementi posljednjeg retka ulaznog vektora sustava odnosno modela prema jednadžbi (3–5),
k_d	-	parametar adaptacije u direktnoj grani
$k_p^T(t) = [k_{p1}(t)\ k_{p2}(t)\ \dots\ k_{pn}(t)]$	-	vektor parametara adaptacije u povratnoj vezi.

$$\underline{\mu}_1 = \underline{i}\,\underline{\psi}_p^T \underline{x} + \underline{i}\,\psi_d u_r,$$
$$\underline{i} = [0\ 0\ \dots\ 0\ 1]^T.$$

Kriterij stabilnosti Ljapunova

Uz sustav oblika

$$V = \underline{e}^T \underline{P}\,\underline{e} + \psi_d \gamma_d^{-1} \psi_d + \underline{\psi}_p^T \underline{\Gamma}_p^{-1} \underline{\psi}_p,$$

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$$\dot{V} = -\underline{e}^T \underline{Q}\,\underline{e} + 2\,\underline{x}^T \underline{\psi}_p \underline{i}^T \underline{P}\,\underline{e} + 2\,\underline{\psi}_p^T \underline{\Gamma}_p^{-1} \underline{\psi}_p +$$
$$+ 2\,u_r \psi_d \underline{i}^T \underline{P}\,\underline{e} + 2\,\psi_d \gamma_d^{-1} \psi_d,$$

$$(\underline{x}^T \underline{\psi}_p)(\underline{i}^T \underline{P}\,\underline{e}) + \underline{\psi}_p^T \underline{\Gamma}_p^{-1} \underline{\psi}_p = 0,$$
$$u_r \psi_d \underline{i}^T \underline{P}\,\underline{e} + \psi_d \gamma_d^{-1} \psi_d = 0$$

$$\dot{\underline{k}}_p^T = \nu \cdot \underline{x}^T \underline{\Gamma}_p, \qquad \dot{k}_d = \nu \cdot u_r \cdot \gamma_d,$$

$\nu = \underline{d}^T \underline{e}$ – *signal poopćene pogreške*

$\underline{d}^T = \dfrac{1}{b_0} \underline{i}^T \underline{P}$ – *vektor težinskih koeficijenata*

Algoritam parametarske adaptacije prema
kriteriju stabilnosti Ljapunova

$$u_s = k_d(t)\,u_r(t) + \underline{k}_p^T(t)\,\underline{x}(t)$$

$$\dot{\underline{k}}_p^T = \nu \cdot \underline{x}^T \Gamma_p, \qquad \dot{k}_d = \nu \cdot u_r \cdot \gamma_d,$$

$\nu = \underline{d}^T \underline{e}$ – *signal poopćene pogreške*

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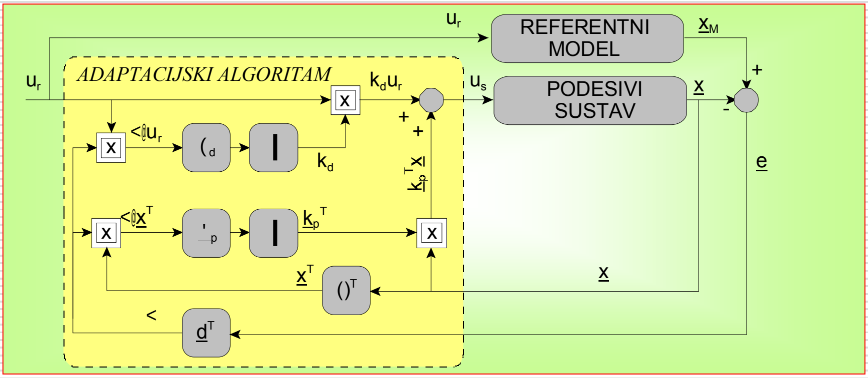
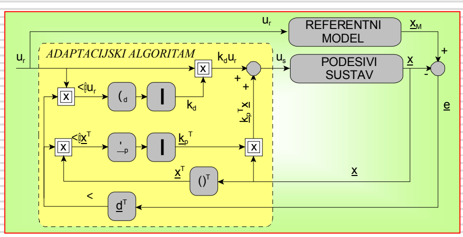


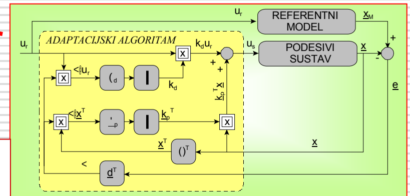
Algoritam parametarske adaptacije prema
kriteriju stabilnosti Ljapunova



- Algoritam adaptacije
  - integralna svojstva
  - varijable stanja
    - prema kanoničkoj osmotrivoj formi
    - potpuni vektor varijabli stanja sustava i modela
- PI ponašanje
  - brži odziv od upotrebe samo I ponašanja
- Adaptacijski algoritam s PI ponašanjem parametara adaptacije
  - Stabilnost – teorija hiperstabilnosti Popova

Algoritam parametarske adaptacije
s PI ponašanjem parametara adaptacije

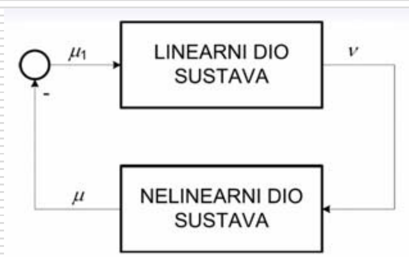
- Adaptacijski algoritam s PI ponašanjem parametara adaptacije



$$\mathbf{k}_\mathbf{p}^T = \mathbf{k}_\mathbf{pi}^T + \mathbf{k}_\mathbf{pp}^T, \qquad k_d = k_{di} + k_{dp},$$
$$\dot{\mathbf{k}}_\mathbf{pi}^T = \nu \cdot \mathbf{x}^T \cdot \mathbf{\Gamma}_\mathbf{pi}, \qquad \dot{k}_{di} = \nu \cdot u_r \cdot \gamma_{di},$$
$$\mathbf{k}_\mathbf{pp}^T = \nu \cdot \mathbf{x}^T \cdot \mathbf{\Gamma}_\mathbf{pp}, \qquad k_{dp} = \nu \cdot u_r \cdot \gamma_{dp},$$
$$\nu = \mathbf{d}^T \mathbf{e},$$

k_{pi}^T	-	redni vektor integralne komponente parametra adaptacije u povratnoj vezi, (dimenzije 1xn),
k_{pp}^T	-	redni vektor proporcionalne komponente parametra adaptacije u povratnoj vezi, (dimenzije 1xn),
k_{di}	-	integralna komponenta parametra adaptacije u direktnoj grani, (dimenzije 1x1),
k_{dp}	-	proporcionalna komponenta parametra adaptacije u direktnoj grani, (dimenzije 1x1),
d^T	-	redni vektor težinskih koeficijenata pogreške,
Γ_{pi}, Γ_{pp}	-	dijagonalne težinske matrice koeficijenata integralnog, odnosno proporcionalnog dijela adaptacijskog algoritma u povratnoj vezi, (dimenzija nxn),
γ_{di}, γ_{dp}	-	koeficijenti integralnog, odnosno proporcionalnog dijela adaptacijskog algoritma u direktnoj grani.

Kriterij hiperstabilnosti Popova



$$\mathrm{Re}\left(G\left(j\omega\right)\right) > 0, \quad \forall \omega > 0$$
$$\int_0^t \boldsymbol{\nu}^T(\tau) \cdot \boldsymbol{\mu}(\tau)\, d\tau \geqslant -\delta, \quad \forall t > 0$$

δ	-	pozitivna konstanta,
μ	-	izlazna varijabla nelinearnog dijela sustava,
ν	-	izlazna varijabla linearnog dijela sustava.

Algoritam parametarske adaptacije
s PI ponašanjem parametara adaptacije

$$\begin{aligned} \mathbf{k}_\mathbf{p}^T &= \mathbf{k}_\mathbf{pi}^T + \mathbf{k}_\mathbf{pp}^T, & k_d &= k_{di} + k_{dp},\\ \dot{\mathbf{k}}_\mathbf{pi}^T &= \nu \cdot \mathbf{x}^T \cdot \mathbf{\Gamma}_\mathbf{pi}, & \dot{k}_{di} &= \nu \cdot u_r \cdot \gamma_{di},\\ \mathbf{k}_\mathbf{pp}^T &= \nu \cdot \mathbf{x}^T \cdot \mathbf{\Gamma}_\mathbf{pp}, & k_{dp} &= \nu \cdot u_r \cdot \gamma_{dp},\\ \nu &= \mathbf{d}^T \mathbf{e}, \end{aligned}$$

$$\dot{\underline{e}} = \underline{A}_M \underline{e} + \underline{\mu}_1 ,$$
$$\nu = \underline{d}^T \underline{e}.$$

Linearni dio sustava

$$\nu(s) = \underline{G}^T(s)\, \underline{\mu}_1(s) = \underline{d}^T(s\underline{I} - \underline{A}_M)^{-1} \underline{\mu}_1(s)$$

Nelinearni dio sustava

$$\underline{\mu} = -\underline{\mu}_1 = (\underline{b}\,\underline{k}_p^T(t,\nu) - (\underline{A}_M - \underline{A}))\underline{x} + (\underline{b}\,k_d(t,\nu) - \underline{b}_M)u_r .$$

Algoritam parametarske adaptacije
s PI ponašanjem parametara adaptacije

Linearni dio sustava

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Nelinearni dio sustava

$$\underline{\mu} = -\underline{\mu}_1 = (\underline{b}\,\underline{k}_p^T(t,\nu) - (\underline{A}_M - \underline{A}))\underline{x} + (\underline{b}\,k_d(t,\nu) - \underline{b}_M)u_r .$$

$$\begin{aligned} \mathbf{k}_\mathbf{p}^T &= \mathbf{k}_\mathbf{pi}^T + \mathbf{k}_\mathbf{pp}^T, & k_d &= k_{di} + k_{dp},\\ \dot{\mathbf{k}}_\mathbf{pi}^T &= \nu \cdot \mathbf{x}^T \cdot \mathbf{\Gamma}_\mathbf{pi}, & \dot{k}_{di} &= \nu \cdot u_r \cdot \gamma_{di},\\ \mathbf{k}_\mathbf{pp}^T &= \nu \cdot \mathbf{x}^T \cdot \mathbf{\Gamma}_\mathbf{pp}, & k_{dp} &= \nu \cdot u_r \cdot \gamma_{dp},\\ \nu &= \mathbf{d}^T \mathbf{e}, \end{aligned}$$

$$\mathrm{Re}\,(G\,(j\omega)) > 0, \quad \forall \omega > 0$$
$$\int_0^t \boldsymbol{\nu}^T(\tau) \cdot \boldsymbol{\mu}(\tau)\, d\tau \geqslant -\delta, \quad \forall t > 0$$

- Linearni dio sustava
  - obuhvaća vektor težinskih koeficijenata pogreške
    - postavljanje nula prijenosne funkcije težinskim koecijentima pogreške
      - pozitivan realni dio prijenosne funkcije linearnog dijela sustava
    - potpuni vektor stanja
- Nelinearni dio sustava
  - obuhvaća algoritam adaptacije

Kriterij hiperstabilnosti Popova

Algoritam s PI ponašanjem parametara adaptacije

Linearni dio sustava

1. uvjet hiperstabilnosti

$$\nu(s) = \underline{G}^T(s)\,\underline{\mu}_1(s) = \underline{d}^T(s\underline{I} - \underline{A}_M)^{-1}\underline{\mu}_1(s)$$

$$\mathrm{Re}\left(G\left(j\omega\right)\right) > 0, \quad \forall\omega > 0$$

- Linearni dio sustava
  - Prijenosna matrica G zadovoljava 1. uvjet hiperstabilnosti Popova ako
    - svaki element matrice zadovoljava tu nejednakost
  - Svaka prijenosna funkcija u prijenosnoj matrici mora biti:
    - stabilna
    - broj polova i nula može se razlikovati najviše za jedan
    - polovi i nule dolaze naizmjenično
      - ukupni fazni kut je u rasponu od -90 do 90 stupnjeva
  - Težinskim koeficijentima matrice d određuje se položaj nula prema polovima.

Kriterij hiperstabilnosti Popova

Algoritam s PI ponašanjem parametara adaptacije

Nelinearni dio sustava

2. uvjet hiperstabilnosti

$$\underline{\mu} = -\underline{\mu}_1 = (\underline{b}\underline{k}_p^T(t,\nu) - (\underline{A}_M - \underline{A}))\underline{x} + (\underline{b}k_d(t,\nu) - \underline{b}_M)u_r.$$

$$\int_0^t \nu(\tau)\cdot\underline{\mu}(\tau)\,d\tau \geq -\underline{\delta}_0,$$

$\underline{\delta}_0 = [\delta_{10}^2\ \delta_{20}^2\ \dots\ \delta_{n0}^2]^T$ - realni vektor pozitivnih konačnih komponenti

- Nelinearni dio sustava
  - Integral umnoška ulaza i izlaza iz nelinearnog dijela sustava
    - ne smije težiti -∞
  - Sve komponente integrala moraju biti
    - konačne ili
    - težiti +∞

Kriterij hiperstabilnosti Popova

Algoritam s PI ponašanjem parametara adaptacije

Nelinearni dio sustava

2. uvjet hiperstabilnosti

$$\underline{\mu} = -\underline{\mu}_1 = (\underline{b}\underline{k}_p^T(t,\nu) - (\underline{A}_M - \underline{A}))\underline{x} + (\underline{b}k_d(t,\nu) - \underline{b}_M)u_r.$$

$$\int_0^t \nu(\tau)\underline{\mu}(\tau)d\tau \geq -\underline{\delta}_0,$$

$$\mathbf{J} = \int_0^t \nu \cdot \left\{ \left[\mathbf{b}\mathbf{k}_\mathbf{p}^T(t,\nu) - \tilde{\mathbf{A}} \right] \mathbf{x} + \left[\mathbf{b}k_d(t,\nu) - \mathbf{b}_\mathbf{M} \right] u_r \right\} d\tau \geqslant -\delta_0$$

$$\tilde{\mathbf{A}} = \mathbf{A}_\mathbf{M} - \mathbf{A}$$

$$\mathbf{J} = \mathbf{J}_1 + \mathbf{J}_2 + \mathbf{J}_3 + \mathbf{J}_4 \geqslant -\delta_0,$$

$$\mathbf{J}_1 = \int_0^t \nu \left(\mathbf{b}\mathbf{k}_{\mathbf{pi}}{}^T(\tau,\nu) - \tilde{\mathbf{A}} \right) \mathbf{x} d\tau, \quad \mathbf{J}_2 = \int_0^t \nu \mathbf{b}\mathbf{k}_{\mathbf{pp}}^T(\tau,\nu)\, \mathbf{x} d\tau,$$

$$\mathbf{J}_3 = \int_0^t \nu \left(\mathbf{b}k_{di}(\tau,\nu) - \mathbf{b}_\mathbf{M} \right) u_r d\tau, \quad \mathbf{J}_4 = \int_0^t \nu \mathbf{b}k_{dp}(\tau,\nu) \cdot u_r d\tau.$$

Kriterij hiperstabilnosti Popova

Algoritam s PI ponašanjem parametara adaptacije

Nelinearni dio sustava 2. uvjet hiperstabilnosti

v – skalar

- komutativnost množenja

$$\dot{\underline{k}}_{pi}^T = \nu \cdot \underline{x}^T \cdot \underline{\Gamma}_{pi},$$

$$\underline{x} \cdot \nu = \underline{\Gamma}_{pi}^{-1} \dot{\underline{k}}_{pi}$$

$$\mathbf{J}_1 = \int_{\mathbf{k}_{\mathbf{pi}}(0)}^{\mathbf{k}_{\mathbf{pi}}(t)} \left(\mathbf{b}\mathbf{k}_{\mathbf{pi}}^T - \tilde{\mathbf{A}} \right) \mathbf{\Gamma}_{\mathbf{pi}}^{-1} d(\mathbf{k}_{\mathbf{pi}}) = \left(\tfrac{1}{2}\mathbf{b}\mathbf{k}_{\mathbf{pi}}^T - \tilde{\mathbf{A}} \right) \mathbf{\Gamma}_{\mathbf{pi}}^{-1}\mathbf{k}_{\mathbf{pi}} \Big|_{\mathbf{k}_{\mathbf{pi}}(0)}^{\mathbf{k}_{\mathbf{pi}}(t)} =$$

$$= \tfrac{1}{2}\mathbf{b}\mathbf{k}_{\mathbf{pi}}^T(t)\, \mathbf{\Gamma}_{\mathbf{pi}}^{-1}\mathbf{k}_{\mathbf{pi}}(t) - \tilde{\mathbf{A}}\mathbf{\Gamma}_{\mathbf{pi}}^{-1}\mathbf{k}_{\mathbf{pi}}(t) -$$

$$- \tfrac{1}{2}\mathbf{b}\mathbf{k}_{\mathbf{pi}}^T(0)\, \mathbf{\Gamma}_{\mathbf{pi}}^{-1}\mathbf{k}_{\mathbf{pi}}(0) + \tilde{\mathbf{A}}\mathbf{\Gamma}_{\mathbf{pi}}^{-1}\mathbf{k}_{\mathbf{pi}}(0)$$

$$\mathbf{J}_2 = \int_0^t \mathbf{b} \cdot \mathbf{k}_{\mathbf{pp}}^T(\tau) \cdot \mathbf{\Gamma}_{\mathbf{pp}}^{-1} \cdot \mathbf{k}_{\mathbf{pp}}(\tau) \cdot d\tau$$

$$\mathbf{J}_4 = \int_0^t \mathbf{b} \cdot k_{dp} \cdot k_{dp} \cdot \gamma_{dp}^{-1} \cdot d\tau = \mathbf{b}\gamma_{dp}^{-1} \int_0^t k_{dp}^2 \cdot d\tau$$

$$\mathbf{J}_3 = \int_0^t (\mathbf{b}k_{di} - \mathbf{b}_\mathbf{M}) \cdot \dot{k}_{di} \cdot \gamma_{di}^{-1} \cdot d\tau = \gamma_{di}^{-1} \left(\tfrac{1}{2}\mathbf{b}k_{di}^2 - \mathbf{b}_\mathbf{M}k_{di} \right) \Big|_{k_{di}(0)}^{k_{di}(t)} =$$

$$= \gamma_{di}^{-1} \left[\tfrac{1}{2}\mathbf{b}k_{di}^2(t) - \mathbf{b}_\mathbf{M}k_{di}(t) - \tfrac{1}{2}\mathbf{b}k_{di}^2(0) + \mathbf{b}_\mathbf{M}k_{di}(0) \right]$$

- J_2 – Umnožak xv izražen pomoću k_{pp}
- J3 – Umnožak u_rv izražen pomoću dk_{di}/dt
- J4 – Umnožak u_rv izražen pomoću k_{dp}

Kriterij hiperstabilnosti Popova

Algoritam s PI ponašanjem parametara adaptacije

Nelinearni dio sustava — **2. uvjet hiperstabilnosti**

$$\mathbf{J}_1 = \tfrac{1}{2}\mathbf{b}\mathbf{k}_{\mathbf{pi}}^T(t)\,\mathbf{\Gamma}_{\mathbf{pi}}^{-1}\mathbf{k}_{\mathbf{pi}}(t) - \tilde{\mathbf{A}}\mathbf{\Gamma}_{\mathbf{pi}}^{-1}\mathbf{k}_{\mathbf{pi}}(t) - -\tfrac{1}{2}\mathbf{b}\mathbf{k}_{\mathbf{pi}}^T(0)\,\mathbf{\Gamma}_{\mathbf{pi}}^{-1}\mathbf{k}_{\mathbf{pi}}(0) + \tilde{\mathbf{A}}\mathbf{\Gamma}_{\mathbf{pi}}^{-1}\mathbf{k}_{\mathbf{pi}}(0)$$

$$\mathbf{J}_3 = \gamma_{di}^{-1}\left[\tfrac{1}{2}\mathbf{b}k_{di}^2(t) - \mathbf{b_M}k_{di}(t) - \tfrac{1}{2}\mathbf{b}k_{di}^2(0) + \mathbf{b_M}k_{di}(0)\right]$$

$$\mathbf{J}_2 = \int_0^t \mathbf{b}\cdot\mathbf{k}_{\mathbf{pp}}^T(\tau)\cdot\mathbf{\Gamma}_{\mathbf{pp}}^{-1}\cdot\mathbf{k}_{\mathbf{pp}}(\tau)\cdot d\tau$$

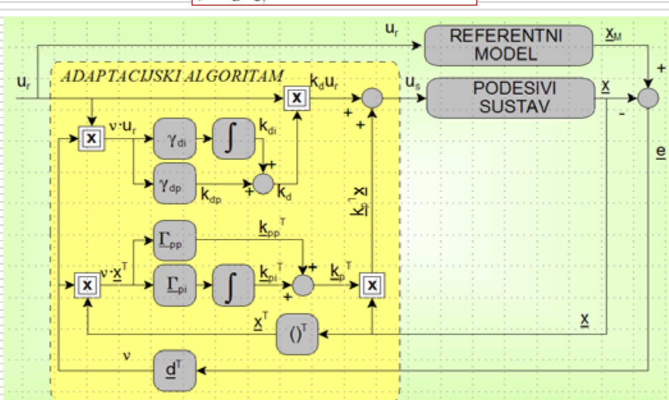
$$\mathbf{J}_4 = \int_0^t \mathbf{b}\cdot k_{dp}\cdot k_{dp}\cdot\gamma_{dp}^{-1}\cdot d\tau = \mathbf{b}\gamma_{dp}^{-1}\int_0^t k_{dp}^2\cdot d\tau$$

- svi integrali imaju vrijednost veću od $-\infty$ za proizvoljne promjene parametara adaptacije, uz uvjet:
  - matrice koeficijenata adaptacije pozitivno definitne
  - matrica $\mathbf{b}$ ima pozitivne retke gdje matrica $\mathbf{A_M} - \mathbf{A}$ ima retke različite od nule
- Analiza
  - $\mathbf{J}_1$ – kvadratna funkcija s pozitivnim koeficijentom uz kvadratni član (postoji minimum)
  - $\mathbf{J}_2$ – integral pozitivne kvadratne funkcije (konačni minimum)
  - $\mathbf{J}_3$ – pozitivna kvadratna funkcija u slučaju da vektor $\mathbf{b}$ ima elemente veće od nule na mjestima gdje vektor $\mathbf{b_M}$ ima nenulte elemente
  - $\mathbf{J}_4$ – integral pozitivne kvadratne funkcije (funkcija s konačnim minimumom)

Shema algoritma adaptacije

Algoritam s PI ponašanjem parametara adaptacije

$$\begin{aligned}
\mathbf{k}_{\mathbf{p}}^T &= \mathbf{k}_{\mathbf{pi}}^T + \mathbf{k}_{\mathbf{pp}}^T, & k_d &= k_{di} + k_{dp},\\
\dot{\mathbf{k}}_{\mathbf{pi}}^T &= \nu\cdot\mathbf{x}^T\cdot\mathbf{\Gamma}_{\mathbf{pi}}, & \dot{k}_{di} &= \nu\cdot u_r\cdot\gamma_{di},\\
\mathbf{k}_{\mathbf{pp}}^T &= \nu\cdot\mathbf{x}^T\cdot\mathbf{\Gamma}_{\mathbf{pp}}, & k_{dp} &= \nu\cdot u_r\cdot\gamma_{dp},\\
\nu &= \mathbf{d}^T\mathbf{e},
\end{aligned}$$



Normiranje signala

- Nivo referentnog signala utječe na brzinu adaptacije
- Parametri adaptacije proporcionalni
  - umnošku vektora pogreške i vektora stanja
  - umnošku vektora pogreške i referentnog signala
- Održanje iste brzine adaptacije u svim radnim točkama
  - ==> NORMIRANJE SIGNALA

Normiranje signala

$$\underline{u}_s = \underline{k}^T \underline{\xi},$$

$$\underline{k}^T = \begin{bmatrix} k_d & \underline{k}_p^T \end{bmatrix} \quad \text{- vektor parametara adaptacije,}$$

$$\underline{\xi} = \begin{bmatrix} u_r \\ \underline{x}_R \end{bmatrix} \quad \text{- vektor signala}$$

Nenormirani parametri adaptacije

$$\underline{k}^T = \underline{k}_i^T + \underline{k}_p^T,$$

$$\underline{\dot{k}}_i^T = \nu \cdot \underline{\xi}^T \cdot \underline{\Gamma}_i,$$

$$\underline{k}_p^T = \nu \cdot \underline{\xi}^T \cdot \underline{\Gamma}_p.$$

$$k_{n0} = \begin{cases} \delta_0 & \forall \left(\dfrac{u_r}{u_{r0}}\right)^2 < \delta_0, \\ \left(\dfrac{u_r}{u_{r0}}\right)^2 & \forall \left(\dfrac{u_r}{u_{r0}}\right)^2 \geq \delta_0. \end{cases}$$

Normirani parametri adaptacije

$$\underline{k}^T = \underline{k}_i^T + \underline{k}_p^T,$$

$$\underline{\dot{k}}_i^T = \frac{1}{k_{n0}} \cdot \nu \cdot \underline{\xi}^T \cdot \underline{\Gamma}_i,$$

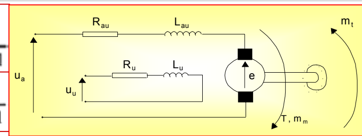
$$\underline{k}_p^T = \frac{1}{k_{n0}} \cdot \nu \cdot \underline{\xi}^T \cdot \underline{\Gamma}_p.$$

Primjer – DC motor

$$u_a = L_{au}\frac{di_a}{dt} + R_{au}i_a + e,$$
$$e = K\omega,$$
$$m_m = Ki_a,$$
$$m = J\frac{d\omega}{dt} + m_t,$$
$$m_t = K_t\omega,$$
$$R_{au} = R_a + R_d, \qquad L_{au} = L_a + L_d$$

$$G_i(s) = \frac{U_i(s)}{I_a(s)} = \frac{K_i}{T_i s + 1}$$

$$G_\omega(s) = \frac{U_\omega(s)}{\Omega(s)} = \frac{K_b}{T_b s + 1}$$



$$I_a(s) = \frac{K_{au}}{T_{au}s + 1}\left(U_a(s) - E(s)\right),$$
$$E(s) = K\Omega(s),$$
$$M_m(s) = KI_a(s),$$
$$\Omega(s) = \frac{1}{Js}\left(M_m(s) - M_t(s)\right),$$
$$M_t(s) = K_t\Omega(s),$$

$$G_{TU}(s) = \frac{U_a(s)}{U_{ri}(s)} = \frac{K_{TU}}{T_{TU}s + 1}$$

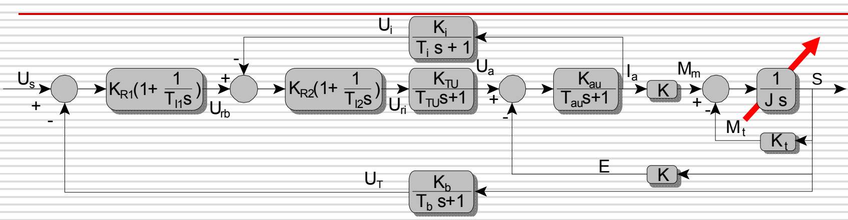
$K_{au}=1/R_{au}$	-	koeficijent pojačanja armaturnog kruga, [1/Ω],
$T_{au}=L_{au}/R_{au}$	-	armaturna vremenska konstanta, [s],
s	-	Laplaceov operator.

i_a	-	struja armature, [A],
J	-	moment inercije motora i tereta, [kg m²],
K	-	konstanta motora, [Vs],
K_t	-	konstanta tereta, [Nms],
L_a	-	induktivitet armaturnog kruga motora, [H],
$L_{au} = L_a + L_d$,		
L_d	-	ekvivalentni induktivitet tiristorskog usmjerivača, [H],
m_m	-	moment motora, [Nm],
m_t	-	moment tereta, [Nm],
R_a	-	radni otpor armature motora, [Ω],
$R_{au} = R_a + R_d$		
R_d	-	ekvivalentni radni otpor tiristorskog usmjerivača, [Ω],
t	-	vrijeme, [t],
u_a	-	armaturni napon, [V],
ω	-	brzina vrtnje, [s⁻¹].

K_{TU}	-	koeficijent pojačanja tiristorskog usmjerivača u kontinuiranom režimu rada.
T_{TU}	-	mrtvo vrijeme tiristorskog usmjerivača, [s]
K_i	-	koeficijent pojačanja mjernog pretvornika struje armature, [V/A],
U_i	-	signal mjerene struje armature, [V],
T_i	-	vremenska konstanta filtera struje armature, [s].
K_b	-	koeficijent pojačanja mjernog pretvornika brzine vrtnje, [Vs],
U_ω	-	signal mjerene brzine vrtnje, [V],
T_b	-	vremenska konstanta filtera signala brzine vrtnje, [s].

Primjer – DC motor

blok shema

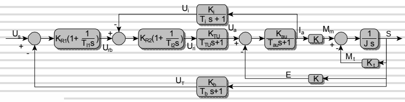


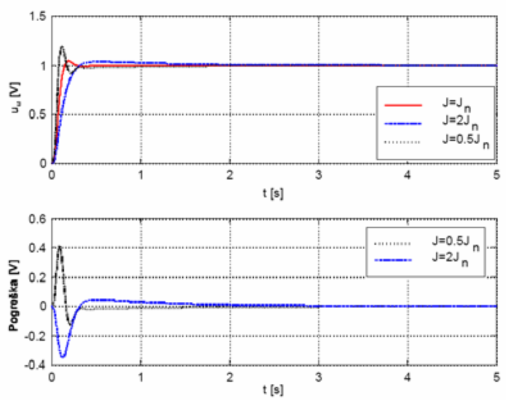
K_{R1}	-	pojačanje PI regulatora brzine vrtnje,
K_{R2}	-	pojačanje PI regulatora struje armature,
$G_{R\Omega}(s)$	-	prijenosna funkcija PI regulatora brzine vrtnje,
$G_{RI}(s)$	-	prijenosna funkcija PI regulatora struje armature,
T_{I1}	-	integralna vremenska konstanta regulatora brzine vrtnje, [s],
T_{I2}	-	integralna vremenska konstanta regulatora struje armature, [s]
$U_s(s)$	-	referentni upravljački signal za elektromotorni pogon,
$U_{rb}(s)$	-	izlazni signal iz regulatora brzine vrtnje,
$U_{ri}(s)$	-	izlazni signal iz regulatora struje armature.

Parametar	Iznos	Jedinica	Parametar	Iznos	Jedinica	Parametar	Iznos	Jedinica
P_n	0.5	kW	K_{au}	0.0612	A/V	T_{au}	18.4	ms
n_n	1500	min⁻¹	K_{TU}	45	V/V	T_{TU}	5	ms
U_{an}	220	V	K_b	0.065	Vs	T_b	25	ms
I_{an}	3.4	A	K_i	1.57	V/A	T_i	5	ms
J_n	0.0157	kg m²	K_{R1}	5	V/V	T_{I1}	1	s
K	1.211	Vs	K_{R2}	0.215	V/V	T_{I2}	18.4	ms
K_t	0.0196	Nms				T_M	801	ms

Primjer – DC motor

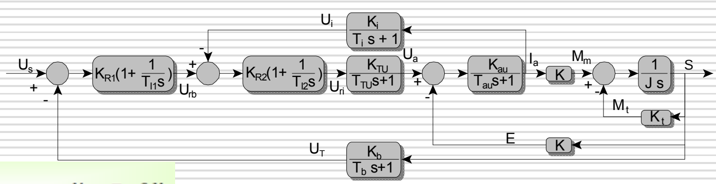
Odziv neadaptivnog sustava





Primjer – DC motor

Prikaz u prostoru stanja



$$\underline{\dot{x}} = \underline{A}\,\underline{x} + \underline{b}\,u_s, \qquad u_\omega = \underline{c}\,\underline{x},$$

$$\underline{x} = \begin{bmatrix} u_{rbI} & u_{riI} & u_a & u_i & i_a & \omega & u_\omega \end{bmatrix}^T$$

$$\underline{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -\frac{K_{R1}}{T_{I1}} \\ \frac{K_{R2}}{T_{I2}} & 0 & 0 & 0 & 0 & 0 & -\frac{K_{R1}K_{R2}}{T_{I2}} \\ \frac{K_{R2}K_{TU}}{T_{TU}} & \frac{K_{TU}}{T_{TU}} & -\frac{1}{T_{TU}} & -\frac{K_{R2}K_{TU}}{T_{TU}} & 0 & 0 & -\frac{K_{R1}K_{R2}K_{TU}}{T_{TU}} \\ 0 & 0 & 0 & -\frac{1}{T_i} & \frac{K_i}{T_i} & 0 & 0 \\ 0 & 0 & \frac{K_{au}}{T_{au}} & 0 & -\frac{1}{T_{au}} & -\frac{K_{au}K}{T_{au}} & 0 \\ 0 & 0 & 0 & 0 & \frac{K}{J} & -\frac{K_t}{J} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{K_b}{T_b} & -\frac{1}{T_b} \end{bmatrix}$$

$$\underline{b} = \begin{bmatrix} \frac{K_{R1}}{T_{I1}} & \frac{K_{R1}K_{R2}}{T_{I2}} & \frac{K_{R1}K_{R2}K_{TU}}{T_{TU}} & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

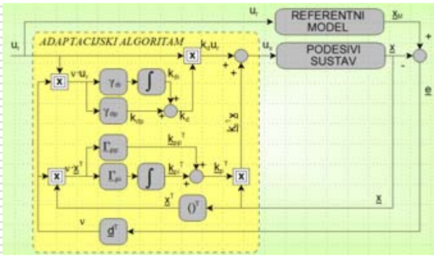
$$\underline{c} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

u_{rbI} - izlaz iz integralnog dijela regulatora brzine vrtnje,

u_{riI} - izlaz iz integralnog dijela regulatora struje armature

Primjer – DC motor

Adaptivni regulator



$$u_s = k_d(t)\, u_r(t) + \underline{k}_p^T(t)\, \underline{x}(t)$$

$$\mathbf{k}_\mathbf{p}^T = \mathbf{k}_\mathbf{pi}^T + \mathbf{k}_\mathbf{pp}^T, \qquad k_d = k_{di} + k_{dp},$$
$$\dot{\mathbf{k}}_\mathbf{pi}^T = \nu \cdot \mathbf{x}^T \cdot \mathbf{\Gamma}_\mathbf{pi}, \qquad \dot{k}_{di} = \nu \cdot u_r \cdot \gamma_{di},$$
$$\mathbf{k}_\mathbf{pp}^T = \nu \cdot \mathbf{x}^T \cdot \mathbf{\Gamma}_\mathbf{pp}, \qquad k_{dp} = \nu \cdot u_r \cdot \gamma_{dp},$$
$$\nu = \mathbf{d}^T \mathbf{e},$$

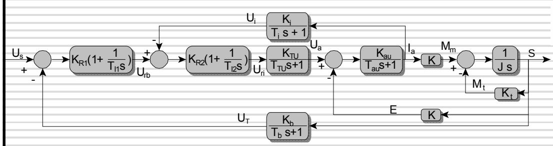
- Vektor varijabli stanja

$$\underline{x} = \begin{bmatrix} u_{rbI} & u_{riI} & u_a & u_i & i_a & \omega & u_\omega \end{bmatrix}^T$$

- Referentni model
  - jednak podesivom sustavu s nominalnim parametrima

$$\dot{\underline{x}}_M = \underline{A}_M \underline{x}_M + \underline{b}_M u_r, \qquad u_{\omega M} = \underline{b}_M \underline{x}_M,$$

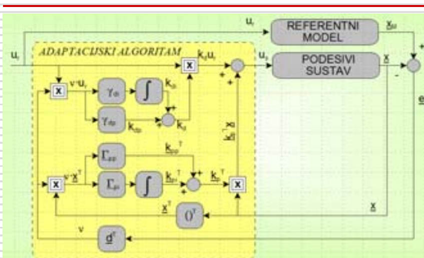
Podesivi sustav

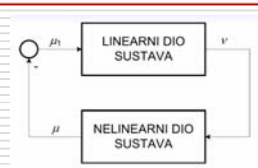


$$\underline{b}_M = \underline{b}, \quad \underline{c}_M = \underline{c}, \quad J_n$$

Primjer – DC motor

Adaptivni regulator – provjera globalne stabilnosti po Popovu





$$\nu(s) = \underline{G}^T(s)\, \underline{\mu}_1(s) = \underline{d}^T(s\underline{I} - \underline{A}_M)^{-1} \underline{\mu}_1(s)$$

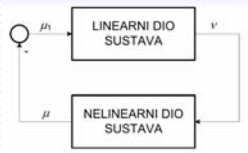
$$\underline{\mu} = -\underline{\mu}_1 = (\underline{b}\underline{k}_p^T(t,\nu) - (\underline{A}_M - \underline{A}))\underline{x} + (\underline{b}k_d(t,\nu) - \underline{b}_M)u_r.$$

- Uvjeti stabilnosti
  - 1. uvjet stabilnosti
    - Re ($\underline{G}^T(j\omega)$) >0
    - $\underline{G}^T$ – redna prijenosna matrica – nazivnik je određen s *det(sI-A$_M$)*,
    - brojnik – nule određene polovima i koeficijentima *d*
    - odabir *d* – zadovoljen 1. uvjet hiperstabilnosti
  - 2. uvjet hiperstabilnosti
    - određivanje integrala

$$\int_0^t \nu(\tau)\cdot\underline{\mu}(\tau)\, d\tau \geq -\underline{\delta}_0,$$

Primjer – DC motor

Adaptivni regulator – provjera globalne stabilnosti po Popovu



$$\nu(s) = \underline{G}^T(s)\,\underline{\mu}_1(s) = \underline{d}^T(s\underline{I} - \underline{A}_M)^{-1}\underline{\mu}_1(s)$$

$$\underline{\mu} = -\underline{\mu}_1 = (\underline{b}\underline{k}_p^T(t,\nu) - (\underline{A}_M - \underline{A}))\underline{x} + (\underline{b}k_d(t,\nu) - \underline{b}_M)u_r.$$

$$\int_0^t \nu(\tau)\cdot\underline{\mu}(\tau)\,d\tau \geq -\underline{\delta}_0,$$

$$\underline{J} = \underline{J}_1 + \underline{J}_2 + \underline{J}_3 + \underline{J}_4 \geq -\underline{\delta}_0, \qquad \tilde{\underline{A}} = \underline{A}_M - \underline{A}$$

$$\underline{J}_1 = \int_0^t \nu(\underline{b}\underline{k}_{pi}^T(\tau,\nu) - \tilde{\underline{A}})\underline{x}\,d\tau, \qquad \underline{J}_2 = \int_0^t \nu\underline{b}\underline{k}_{pp}^T(\tau,\nu)\underline{x}\,d\tau,$$

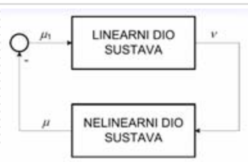
$$\underline{J}_3 = \int_0^t \nu(\underline{b}k_{di}(\tau,\nu) - \underline{b}_M)u_r\,d\tau, \qquad \underline{J}_4 = \int_0^t \nu\underline{b}k_{dp}(\tau,\nu)\cdot u_r\,d\tau.$$

$$\Gamma_{PI} = \begin{bmatrix} 0 & \gamma_{PI2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_{PI3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_{PI4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma_{PI5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma_{PI6} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \gamma_{PI7} \end{bmatrix}, \qquad K_{PI} = \begin{bmatrix} k_{PI2}(t) \\ k_{PI3}(t) \\ k_{PI4}(t) \\ k_{PI5}(t) \\ k_{PI6}(t) \\ k_{PI7}(t) \end{bmatrix},$$

$$\tilde{\underline{A}} = \underline{A}_M - \underline{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K\left(\frac{1}{J_n} - \frac{1}{J}\right) & -K_I\left(\frac{1}{J_n} - \frac{1}{J}\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Primjer – DC motor

Adaptivni regulator – provjera globalne stabilnosti po Popovu



$$\underline{J} = \underline{J}_1 + \underline{J}_2 + \underline{J}_3 + \underline{J}_4 \geq -\underline{\delta}_0, \qquad \tilde{\underline{A}} = \underline{A}_M - \underline{A}$$

$$\underline{J}_1 = \int_0^t \nu(\underline{b}\underline{k}_{pi}^T(\tau,\nu) - \tilde{\underline{A}})\underline{x}\,d\tau, \qquad \underline{J}_2 = \int_0^t \nu\underline{b}\underline{k}_{pp}^T(\tau,\nu)\underline{x}\,d\tau,$$

$$\underline{J}_3 = \int_0^t \nu(\underline{b}k_{di}(\tau,\nu) - \underline{b}_M)u_r\,d\tau, \qquad \underline{J}_4 = \int_0^t \nu\underline{b}k_{dp}(\tau,\nu)\cdot u_r\,d\tau.$$

Kvadratne funkcije koeficijenata K_{PI} – imaju minimum

$$\underline{J}_1 = \begin{bmatrix} \frac{K_{RI}}{2T_{I1}}\left(k_{PI1}^2(t)\gamma_{PI1}^{-1} + k_{PI2}^2(t)\gamma_{PI2}^{-1} + k_{PI3}^2(t)\gamma_{PI3}^{-1} + k_{PI4}^2(t)\gamma_{PI4}^{-1} + k_{PI5}^2(t)\gamma_{PI5}^{-1} + k_{PI6}^2(t)\gamma_{PI6}^{-1} + k_{PI7}^2(t)\gamma_{PI7}^{-1}\right) + c_1 \\ \frac{K_{RI}K_{R2}}{2T_{I2}}\left(k_{PI1}^2(t)\gamma_{PI1}^{-1} + k_{PI2}^2(t)\gamma_{PI2}^{-1} + k_{PI3}^2(t)\gamma_{PI3}^{-1} + k_{PI4}^2(t)\gamma_{PI4}^{-1} + k_{PI5}^2(t)\gamma_{PI5}^{-1} + k_{PI6}^2(t)\gamma_{PI6}^{-1} + k_{PI7}^2(t)\gamma_{PI7}^{-1}\right) + c_2 \\ \frac{K_{RI}K_{R2}ktu}{2T_{TU}}\left(k_{PI1}^2(t)\gamma_{PI1}^{-1} + k_{PI2}^2(t)\gamma_{PI2}^{-1} + k_{PI3}^2(t)\gamma_{PI3}^{-1} + k_{PI4}^2(t)\gamma_{PI4}^{-1} + k_{PI5}^2(t)\gamma_{PI5}^{-1} + k_{PI6}^2(t)\gamma_{PI6}^{-1} + k_{PI7}^2(t)\gamma_{PI7}^{-1}\right) + c_3 \\ 0 \\ 0 \\ \ldots - \frac{K}{J})\gamma_{PI5}^{-1}k_{PI5}(t) - (-\frac{K_I}{J_n} + \frac{K_I}{J})\gamma_{PI6}^{-1}k_{PI6}(t) + c_4 \\ 0 \end{bmatrix}$$

- Linearna funkcija koeficijenjta K_{PI}
- nije zagarantiran minimum
- Globalna stabilnost
  - nije zagarantirana
- postoji samo lokalna stabilnost

Primjer – DC motor – drugi odabir varijabli stanja

Adaptivni regulator – provjera globalne stabilnosti po Popovu

$$G(s)=\frac{u_\omega(s)}{u_s(s)}=\frac{b_3s^3+b_2s^2+b_1s+b_0}{s^7+a_6s^6+a_5s^5+a_4s^4+a_3s^3+a_2s^2+a_1s+a_0}$$

$$\dot{\underline{x}}=\underline{A}\underline{x}+\underline{b}u_s,\qquad u_\omega=\underline{c}\underline{x},$$

$$\underline{A}=\begin{bmatrix}0&1&0&0&0&0&0\\0&0&1&0&0&0&0\\0&0&0&1&0&0&0\\0&0&0&0&1&0&0\\0&0&0&0&0&1&0\\0&0&0&0&0&0&1\\-a_0&-a_1&-a_2&-a_3&-a_4&-a_5&-a_6\end{bmatrix},\quad \underline{b}=\begin{bmatrix}0\\0\\0\\b_3\\-a_6b_3+b_2\\(-a_5+a_6^2)b_3-a_6b_2+b_1\\(-a_4+2a_5a_6-a_6^3)b_3+(-a_5+a_6^2)b_2-a_6b_1+b_0\end{bmatrix}$$

$$\underline{c}=[1\ 0\ 0\ 0\ 0\ 0\ 0]$$

Referentni model jednak sustavu s nominalnim parametrima potpuni vektor stanja

Zadovoljen kriterij hiperstabilnosti
elementi matrice b pozitivni za moment inercije 0# J# $10^7 J_n$

$$\mathbf{J}_1=\tfrac{1}{2}\mathbf{b}\mathbf{k}_{\mathbf{pi}}^T(t)\,\mathbf{\Gamma}_{\mathbf{pi}}^{-1}\mathbf{k}_{\mathbf{pi}}(t)-\tilde{\mathbf{A}}\mathbf{\Gamma}_{\mathbf{pi}}^{-1}\mathbf{k}_{\mathbf{pi}}(t)-$$
$$-\tfrac{1}{2}\mathbf{b}\mathbf{k}_{\mathbf{pi}}^T(0)\,\mathbf{\Gamma}_{\mathbf{pi}}^{-1}\mathbf{k}_{\mathbf{pi}}(0)+\tilde{\mathbf{A}}\mathbf{\Gamma}_{\mathbf{pi}}^{-1}\mathbf{k}_{\mathbf{pi}}(0)$$

$$\mathbf{J}_3=\gamma_{di}^{-1}\left[\tfrac{1}{2}\mathbf{b}k_{di}^2(t)-\mathbf{b}_\mathbf{M}k_{di}(t)-\tfrac{1}{2}\mathbf{b}k_{di}^2(0)+\mathbf{b}_\mathbf{M}k_{di}(0)\right]$$

$$\mathbf{J}_2=\int_0^t\mathbf{b}\cdot\mathbf{k}_{\mathbf{pp}}^T(\tau)\cdot\mathbf{\Gamma}_{\mathbf{pp}}^{-1}\cdot\mathbf{k}_{\mathbf{pp}}(\tau)\cdot d\tau$$
$$\mathbf{J}_4=\int_0^t\mathbf{b}\cdot k_{dp}\cdot k_{dp}\cdot\gamma_{dp}^{-1}\cdot d\tau=\mathbf{b}\gamma_{dp}^{-1}\int_0^t k_{dp}^2\cdot d\tau$$

- ❑Problem
- ❑Varijable stanja procesa
  - ❑nisu mjerljive

Algoritam adaptacije

s reduciranim vektorom varijabli stanja

$$\dot{\underline{x}}_M=\underline{A}_M\underline{x}_M+\underline{b}_Mu_r,$$
$$\dot{\underline{x}}=\underline{A}\underline{x}+\underline{b}u_s,$$

$$\underline{x}_{MR}=\underline{F}\underline{x}_M,$$
$$\underline{x}_R=\underline{F}\underline{x},$$

$$\underline{e}_R=\underline{x}_{MR}-\underline{x}_R=\underline{F}\underline{e},$$

$$\nu=\underline{d}^T\cdot(\underline{x}_{MR}-\underline{x}_R)=\underline{d}^T\underline{F}\underline{e}=\underline{d}^T\underline{e}_R,$$
$$u_s=k_du_r+\underline{k}_p^T\underline{x}_R=$$
$$=(k_{dp}+k_{di})u_r+(\underline{k}_{pp}^T+\underline{k}_{pi}^T)\underline{F}\underline{x}.$$

$$\underline{k}_p^T=\underline{k}_{pi}^T+\underline{k}_{pp}^T,\qquad k_d=k_{di}+k_{dp},$$
$$\dot{\underline{k}}_{pi}^T=\nu\cdot\underline{x}_R^T\cdot\underline{\Gamma}_{pi},\qquad \dot{k}_{di}=\nu\cdot u_r\cdot\gamma_{di},$$
$$\underline{k}_{pp}^T=\nu\cdot\underline{x}_R^T\cdot\underline{\Gamma}_{pp},\qquad k_{dp}=\nu\cdot u_r\cdot\gamma_{dp},$$

- $\underline{e}_R$ - reducirani vektor pogreške,
- $\underline{x}_{MR}$ - reducirani vektor varijabli stanja referentnog modela dimenzije *m*, manje od reda referentnog modela,
- $\underline{x}_R$ - reducirani vektor varijabli stanja podesivog sustava dimenzije *m*, manje od reda podesivog sustava,
- $\underline{F}$ - matrica transformacije reduciranog vektora varijabli stanja dimenzije *mxn*, (gdje je n red sustava, a m red reduciranog vektora stanja).

Primjer F 1. red

$$\underline{F}=[1\ 0\ 0\ 0\ 0\ 0\ 0]$$
$$\underline{d}=d_1\ .$$

Primjer F 3. red

$$\underline{F}=\begin{bmatrix}1&0&0&0&0&0&0\\0&1&0&0&0&0&0\\0&0&1&0&0&0&0\end{bmatrix},$$
$$\underline{d}^T=[d_1\ d_2\ d_3]\ .$$

Algoritam adaptacije

s reduciranim vektorom varijabli stanja - STABILNOST

$$\nu = \underline{d}^T \cdot (\underline{x}_{MR} - \underline{x}_R) = \underline{d}^T \underline{E}\underline{e} = \underline{d}^T \underline{e}_R,$$

$$u_s = k_d u_r + \underline{k}_p^T \underline{x}_R = (k_{dp} + k_{di}) u_r + (\underline{k}_{pp}^T + \underline{k}_{pi}^T)\underline{E}\underline{x}.$$

$$\dot{\underline{e}} = \underline{A}_M \underline{e} + \underline{\mu}_1,$$

$$\nu = \underline{d}^T \underline{E}\underline{e} = \underline{d}^T \underline{e}_R,$$

$$\underline{\mu} = -\underline{\mu}_1 = \left[\underline{b}\left(\underline{k}_{pp}^T + \underline{k}_{pi}^T\right)\underline{E} - \tilde{\underline{A}}\right]\underline{x} + \left[\underline{b}\left(k_{dp} + k_{di}\right) - b_M\right]u_r$$

$$\underline{G}_R^T(s) = \underline{d}^T \underline{E}(s\underline{I} - \underline{A}_M)^{-1}$$

$$\underline{J} = \int_0^t \nu\left\{\left[\underline{b}(\underline{k}_{pp}^T + \underline{k}_{pi}^T)\underline{E} - \tilde{\underline{A}}\right]\underline{x} + \left[\underline{b}(k_{dp} + k_{di}) - \underline{b}_M\right]u_r\right\}d\tau \geq -\underline{\delta}_0.$$

$$\underline{J} = \underline{J}_1 + \underline{J}_2 + \underline{J}_3 + \underline{J}_4 \geq -\underline{\delta}_0,$$

$$\underline{J}_1 = \int_0^t \nu(\underline{b}\underline{k}_{pi}^T \underline{E} - \tilde{\underline{A}})\underline{x}\,d\tau, \qquad \underline{J}_2 = \int_0^t \nu\underline{b}\underline{k}_{pp}^T \underline{E}\underline{x}\,d\tau,$$

$$\underline{J}_3 = \int_0^t \nu(\underline{b}k_{di} - \underline{b}_M)u_r\,d\tau, \qquad \underline{J}_4 = \int_0^t \nu\underline{b}k_{dp}\,u_r\,d\tau.$$

Algoritam adaptacije

s reduciranim vektorom varijabli stanja - STABILNOST

$$\underline{G}_R^T(s) = \underline{d}^T \underline{E}(s\underline{I} - \underline{A}_M)^{-1}$$

$$\underline{J} = \underline{J}_1 + \underline{J}_2 + \underline{J}_3 + \underline{J}_4 \geq -\underline{\delta}_0,$$

$$\underline{J}_1 = \int_0^t \nu(\underline{b}\underline{k}_{pi}^T \underline{E} - \tilde{\underline{A}})\underline{x}\,d\tau, \qquad \underline{J}_2 = \int_0^t \nu\underline{b}\underline{k}_{pp}^T \underline{E}\underline{x}\,d\tau,$$

$$\underline{J}_3 = \int_0^t \nu(\underline{b}k_{di} - \underline{b}_M)u_r\,d\tau, \qquad \underline{J}_4 = \int_0^t \nu\underline{b}k_{dp}\,u_r\,d\tau.$$

$$\underline{k}_{pi}^T(t) = \int_0^t \nu\underline{x}^T \underline{E}^T \underline{\Gamma}_{pi}\,d\tau + \underline{k}_{pi}^T(0) = \underline{J}_x^T(t)\cdot\underline{E}^T\cdot\underline{\Gamma}_{pi} + \underline{k}_{pi}^T(0)$$

$$\underline{J}_1 = \underline{b}\int_0^t \underline{k}_{pi}^T \underline{E}\nu\underline{x}\,d\tau - \tilde{\underline{A}}\int_0^t \nu\underline{x}\,d\tau =$$

$$= \underline{b}\int_0^t \left[\underline{J}_x^T(\tau)\cdot\underline{E}^T\cdot\underline{\Gamma}_{pi} + \underline{k}_{pi}^T(0)\right]\underline{E}\cdot\dot{\underline{J}}_x(\tau)\,d\tau - \tilde{\underline{A}}\cdot\underline{J}_x(t) =$$

$$= \frac{1}{2}\underline{b}\cdot\underline{J}_x^T(t)\underline{E}^T\underline{\Gamma}_{pi}\underline{E}\underline{J}_x(t) + (\underline{b}\underline{k}_{pi}^T(0)\underline{E} - \tilde{\underline{A}})\underline{J}_x(t) -$$

$$- \frac{1}{2}\underline{b}\cdot\underline{J}_x^T(0)\underline{E}^T\underline{\Gamma}_{pi}\underline{E}\underline{J}_x(0) - \underline{b}\underline{k}_{pi}^T(0)\underline{E}\underline{J}_x(0),$$

1. uvjet hiperstabilnosti zadovoljen
   - ako je prijenosna funkcije referentnog modela stabilna i ako joj relativni red (razlika reda nazivnika i brojnika prijenosne funkcije) nije veći od m, gdje je m broj varijabli stanja koje se koriste u adaptacijskom algoritmu
   - pravilan izbor konstanti težinske matrice $\underline{d}^T$ može se postići da prijenosna funkcija linearnog dijela zadovoljava uvjet hiperstabilnosti

- integrali J_3 i J_4 identični su kao kod primjene potpunog vektora varijabli stanja
- integral J_1 zadovoljava drugi uvjet hiperstabilnosti uz uvjet da:
  - je dijagonalna matrica težinskih koeficijenata $\underline{\Gamma}_{pi}$ pozitivno definitna
  - matrica $\underline{b}$ ima elemente veće od nule recima gdje matrica $(\underline{b}\underline{k}_{pi}^T(0)\underline{E} - \tilde{\underline{A}})$ ima retke različite od nule
  - elementi ostalih redaka matrice $\underline{b}$ budu veći ili jednaki nuli
- integral J_2 zadovoljava drugi uvjet hiperstabilnosti
  - izražavanjem umnoška poopćene pogreške i reduciranog vektora stanja koeficijentom adaptacije, dobije se isti izraz kao kod integrala za potpuni vektor stanja

Primjer DC motor

Algoritam adaptacije s reduciranim vektorom varijabli stanja

$$\underline{F} = [1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0],\quad \underline{d} = d_1 .$$

- vektor stanja 1. reda
  - nezadovoljen 1. uvjet hiperstabilnosti
    - (relativni red prijenosne funkcije linearnog dijela sustava 6.)

$$\underline{F} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},\quad \underline{d}^T = [d_1\ \ d_2] .$$

- vektor stanja 2. reda
  - nezadovoljen 1. uvjet hiperstabilnosti
    - (relativni red prijenosne funkcije linearnog dijela sustava 5.)

$$\underline{F} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix},\quad \underline{d}^T = [d_1\ \ d_2\ \ d_3] .$$

- vektor stanja 3. reda
  - nezadovoljen 1. uvjet hiperstabilnosti
    - (relativni red prijenosne funkcije linearnog dijela sustava 4.)

- moguća samo lokalna stabilnost
- potrebno istražiti simuliranjem

Algoritam adaptacije

s referentnim modelom reduciranog reda

$$\dot{\underline{x}}_{MR} = \underline{A}_{MR}\underline{x}_{MR} + \underline{b}_{MR}u_r,$$

$$\dot{\underline{x}} = \underline{A}\underline{x} + \underline{b}u_s,$$

$$\underline{x}_R = \underline{F}\underline{x},$$

- $\underline{A}$ - matrica stanja podesivog sustava dimenzije nxn,
- $\underline{A}_{MR}$ - matrica stanja reduciranog referentnog modela dimenzije mxm ($m \leq n$),
- $\underline{b}$ - ulazna matrica podesivog sustava dimenzije $nx1$,
- $\underline{b}_{MR}$ - ulazna matrica reduciranog referentnog modela dimenzije $mx1$,
- $\underline{F}$ - matrica transformacije vektora stanja podesivog sustava dimenzije mxn,
- $\underline{x}$ - vektor stanja podesivog sustava dimenzije $nx1$,
- $\underline{x}_R$ - reducirani vektor stanja podesivog sustava dimenzije $mx1$,
- $\underline{x}_{MR}$ - vektor stanja reduciranog referentnog modela dimenzije $mx1$.

$$\boxed{\dot{\underline{e}}_R = \underline{A}_{MR}\underline{e}_R + (\underline{A}_{MR}\underline{F} - \underline{F}\underline{A})\underline{x} + \underline{b}_{MR}u_r - \underline{F}\underline{b}u_s}$$

$$\nu = \underline{d}^T\underline{e}_R = \underline{d}^T(\underline{x}_{MR} - \underline{x}_R) ,$$

$$\underline{\mu} = -\underline{\mu}_1 = \left[\underline{F}\underline{A} - \underline{A}_{MR}\underline{F} + \underline{F}\underline{b}\left(\underline{k}_{pp}^T + \underline{k}_{pi}^T\right)\underline{F}\right]\underline{x} + \left[\underline{F}\underline{b}\left(k_{dp} + k_{di}\right) - b_{MR}\right]u_r$$

$$\boxed{\underline{G}^T(s) = \underline{d}^T(s\underline{I} - \underline{A}_{MR})^{-1}}$$

$$\underline{\tilde{A}}_R = \underline{A}_{MR}\underline{F} - \underline{F}\underline{A}.$$

$$\boxed{\underline{J} = \int_0^t \nu\left\{\left[\underline{F}\underline{b}\left(\underline{k}_{pp}^T + \underline{k}_{pi}^T\right)\underline{F} - \underline{\tilde{A}}_R\right]\underline{x} + \left[\underline{F}\underline{b}\left(k_{dp} + k_{di}\right) - b_{MR}\right]u_r\right\}d\tau \geq -\underline{\delta}_0^2,}$$

Primjer DC motor

Algoritam adaptacije s referentnim modelom reduciranog reda

$$G_{MR}(s) = \frac{u_{MR\omega}}{u_r} = \frac{b_{M0}}{a_{M3}s^3 + a_{M2}s^2 + a_{M1}s + a_{M0}},$$

Model	b_{M0}	a_{M0}	a_{M1}	a_{M2}	a_{M3}	e_{max} [%]
3. red	3.0968e+004	3.0968e+004	2.2045e+003	7.5634e+001	1.0000e+000	1.49

- Koeficijenti referentnog modela određeni optimiranjem
  - prema sustavu s nominalnim parametrima
  - ISE kriterij

$$I_{ISE} = \int_0^t (u_{MR\omega} - u_\omega)^2 d\tau,$$

Prijenosna matrica linearnog dijela

$$\underline{G}^T_{MR3}(s) = \begin{bmatrix} \dfrac{d_1 s^2 + (d_1 a_{M2} - d_3 a_{M0})s + d_1 a_{M1} - d_2 a_{M0}}{s^3 + a_{M2}s^2 + a_{M1}s + a_{M0}} \\ \dfrac{d_2 s^2 + (d_1 + a_{M2}d_2 - d_3 a_{M1})s + d_1 a_{M2} - d_3 a_{M0}}{s^3 + a_{M2}s^2 + a_{M1}s + a_{M0}} \\ \dfrac{d_3 s^2 + d_2 s + d_1}{s^3 + a_{M2}s^2 + a_{M1}s + a_{M0}} \end{bmatrix}$$

Polovi prijenosne matrice

$$\underline{p} = \begin{bmatrix} -39.7889 \\ -17.9226 + j21.3794 \\ -17.9226 - j21.3794 \end{bmatrix}$$

$$\underline{\mu} = -\underline{\mu}_1 = \begin{bmatrix} 0 \\ 0 \\ a_{M0}x_1 + a_{M1}x_2 + a_{M2}x_3 + x_4 - u_r b_{M0} \end{bmatrix}.$$

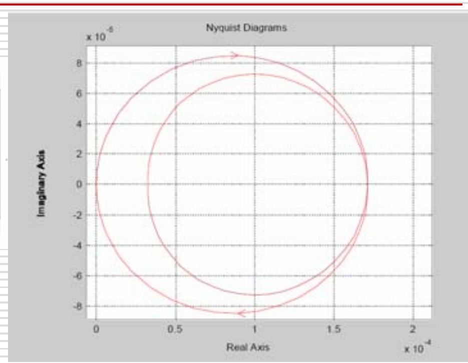
Odabir nula na 0.4 realnog dijla pola za 3. prijenosnu funkciju (samo je ona pobuđena vektorom μ):
z1=-15,9155 z2=-7,1691

Primjer DC motor

Algoritam adaptacije s referentnim modelom reduciranog reda

Prijenosna matrica linearnog dijela

$$\underline{G}^T_{MR3}(s) = \begin{bmatrix} \dfrac{d_1 s^2 + (d_1 a_{M2} - d_3 a_{M0})s + d_1 a_{M1} - d_2 a_{M0}}{s^3 + a_{M2}s^2 + a_{M1}s + a_{M0}} \\ \dfrac{d_2 s^2 + (d_1 + a_{M2}d_2 - d_3 a_{M1})s + d_1 a_{M2} - d_3 a_{M0}}{s^3 + a_{M2}s^2 + a_{M1}s + a_{M0}} \\ \dfrac{d_3 s^2 + d_2 s + d_1}{s^3 + a_{M2}s^2 + a_{M1}s + a_{M0}} \end{bmatrix}$$



- Koeficijenti $\underline{d}$ određeni prema nulama 3. prijenosne funkcije
- Ostali parametri određeni optimiranjem prema ISE kriteriju

Odabir nula na 0.4 realnog dijla pola za 3. prijenosnu funkciju:
z1=-15,9155 z2=-7,1691

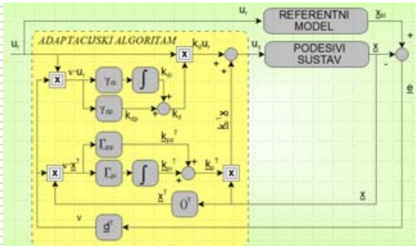
$$d_3 s^2 + d_2 s + 1 = \left(\frac{1}{z_1}s - 1\right)\left(\frac{1}{z_2}s - 1\right)$$

d1=1
d2=0.20232
d3=8.7643·10^{-3}

Ostali koeficijenti optimiranje po ISE kriteriju

Primjer DC motor

Algoritam adaptacije s referentnim modelom reduciranog reda



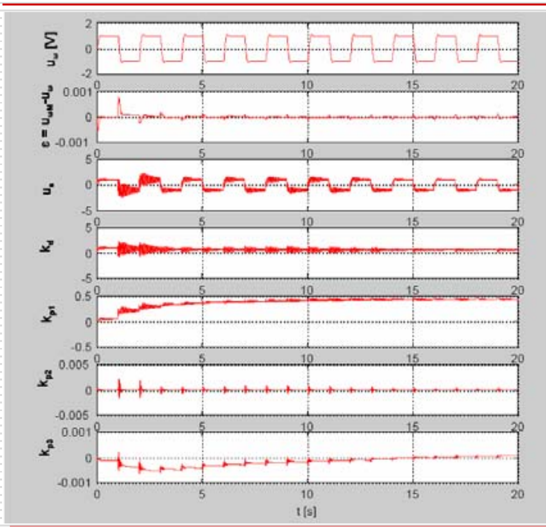
$$G_{MR}(s) = \frac{u_{MRo}}{u_r} = \frac{b_{M0}}{a_{M3}s^3 + a_{M2}s^2 + a_{M1}s + a_{M0}},$$

Model	b_{M0}	a_{M0}	a_{M1}	a_{M2}	a_{M3}	[illegible] [%]
3. red	3.0968e+004	3.0968e+004	2.2045e+003	7.5634e+001	1.0000e+000	1.49

Rezultati optimiranja						Rezultati simuliranja		
J/J_n	d_1 d_2 d_3	γ_{dp}	γ_{di}	γ_{pp1} γ_{pp2} γ_{pp3}	γ_{pi1} γ_{pi2} γ_{pi3}	J/J_n	ε_{mo} [%]	ε_{ms} [%]
0.5	1 0.20232 $8.7643 \cdot 10^{-3}$	7.1293	589.6935	0.4299 0 0	189.8297 0 0	0.5	0.2147	0.0224
						2.0	0.3986	0.0833

Primjer DC motor

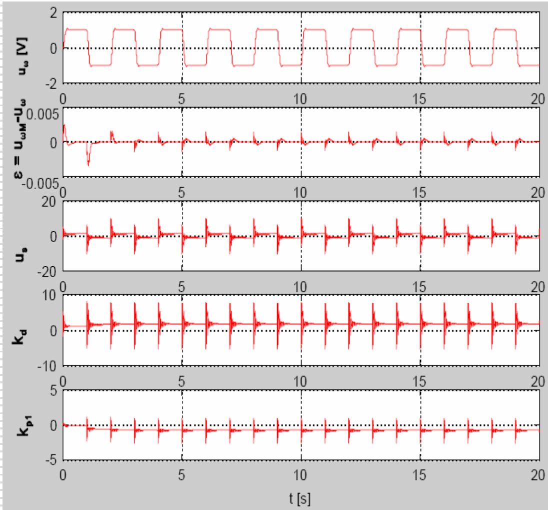
Algoritam adaptacije s referentnim modelom reduciranog reda



J/Jn=0.5
koeficijenti adaptacije optimirani za J/Jn=0.5

Primjer DC motor

Algoritam adaptacije s referentnim modelom reduciranog reda



J/Jn=2
koeficijenti adaptacije
optimirani za J/Jn=0.5

Primjer DC motor

Algoritam adaptacije s referentnim modelom reduciranog reda

Iznos maksimalne pogreške neadaptivnog sustava
e=41.1% za J=0.5J$_n$
e=35.2% za J=2J$_n$.

$$G_{MR}(s) = \frac{u_{MR\omega}}{u_r} = \frac{b_{M0}}{a_{M3}s^3 + a_{M2}s^2 + a_{M1}s + a_{M0}},$$

Model	b_{M0}	a_{M0}	a_{M1}	a_{M2}	a_{M3}	e_{max} [%]
3. red	3.0968e+004	3.0968e+004	2.2045e+003	7.5634e+001	1.0000e+000	1.49

Adaptivni sustav

Rezultati optimiranja						Rezultati simuliranja		
J/J_n	d_1 d_2 d_3	γ_{dp}	γ_{di}	γ_{pp1} γ_{pp2} γ_{pp3}	γ_{pi1} γ_{pi2} γ_{pi3}	J/J_n	ε_{mo} [%]	ε_{ms} [%]
0.5	1 0.20232 $8.7643 \cdot 10^{-3}$	7.1293	589.6935	0.4299 0 0	89.8297 0 0	0.5	0.2147	0.0224
						2.0	0.3986	0.0833