Predicting the Electricity Demand Response via Data-driven Inverse Optimization

Workshop on Demand Response and Energy Storage Modeling
Zagreb, Croatia

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June 19$^{\text{th}}$, 2018

European Research Council
Established by the European Commission

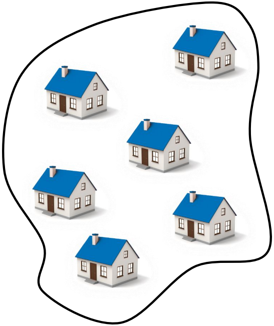
Optimization and Analytics for Sustainable energY Systems

Outline

- Motivation
- Forecasting the price-responsive costumers' demand
- Defining the estimation problem
- Solving the estimation problem
- Case studies
  - One-hour ahead prediction: Simulated pool of buildings with HVAC systems
  - One-day head prediction: Real-life experiment
- Conclusions

Assumptions

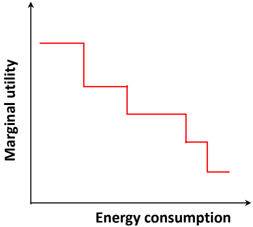
A cluster of **price-responsive** consumers is considered



This cluster is expected to **consume** more **at a favorable price**

We describe the pool of price-responsive consumers as a **utility maximizer agent**

Step-wise marginal utility function



Consumers' price-response model

$$
\begin{aligned}
\underset{x_{b,t},\forall b,t}{\text{maximize}} \quad & \sum_{b=1}^{B} x_{b,t}(u_{b,t} - p_t) \\
\text{subject to} \quad & \underline{P}_t \leq \sum_{b=1}^{B} x_{b,t} \leq \overline{P}_t, \quad \forall t \qquad (\underline{\lambda}_t, \overline{\lambda}_t) \\
& 0 \leq x_{b,t} \leq E_b, \quad \forall t \qquad (\underline{\phi}_{b,t}, \overline{\phi}_{b,t})
\end{aligned}
$$

It is a **linear optimization problem (LOP)**.

Unknown variables:

- **Marginal utilities** $u_{b,t}$
- **Power bounds** $\overline{P}_t, \underline{P}_t$

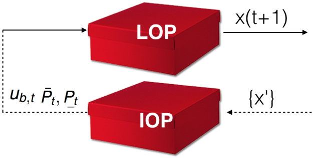
We seek values of $u_{b,t}$, $\overline{P}_t$, and $\underline{P}_t$ based on observations of $x'_{b,t}$ and p_t, given E_b. We use the estimated utility maximizer problem to predict x_{t+1}.

The estimation problem: Optimality condition

$$
\begin{aligned}
\underset{\Omega}{\text{minimize}} \quad & \sum_{t=1}^{T} \epsilon_t \\
\text{subject to} \quad & \overline{P}_t\overline{\lambda}_t - \underline{P}_t\underline{\lambda}_t + \sum_{b=1}^{B} E_b\overline{\phi}_{b,t} - \epsilon_t = \sum_{b=1}^{B} x_{b,t}(u_{b,t} - p_t) \\
& \overline{\phi}_{b,t} - \underline{\phi}_{b,t} + \overline{\lambda}_t - \underline{\lambda}_t = u_{b,t} - p_t \\
& \overline{\phi}_{b,t}, \underline{\phi}_{b,t}, \overline{\lambda}_t, \underline{\lambda}_t, \epsilon_t \geq 0
\end{aligned}
$$

$$\Omega = \left\{\epsilon_t, \overline{P}_t, \underline{P}_t, u_{b,t}, \overline{\lambda}_t, \underline{\lambda}_t, \overline{\phi}_{b,t}, \underline{\phi}_{b,t}\right\}$$

Inverse optimization (IOP) is used to determine the **parameters** of the model to make **predictions** of the load.



Leveraging auxiliary information

Model parameters $\overline{P}_t$, $\underline{P}_t$ and $u_{b,t}$, might vary over time. We assume a number of time varying regressors Z such that

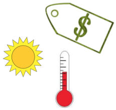
$$\underline{P}_t = \underline{\mu} + \sum_{r=1}^{R} \underline{\alpha}_r Z_{r,t} \tag{1}$$

$$\overline{P}_t = \overline{\mu} + \sum_{r=1}^{R} \overline{\alpha}_r Z_{r,t} \tag{2}$$

$$u_{b,t} = \mu_b^u + \sum_{r=1}^{R} \alpha_r^u Z_{r,t} \tag{3}$$

Regressors relate to **time** and **weather**:

- Temperature of the air outside
- Solar irradiance
- Hour indicator
- Past price and load



Leveraging auxiliary information

The bid must make sense for any plausible value of the features, in particular,

- The minimum consumption limit must be lower than or equal to the maximum consumption limit
- The minimum consumption limit must be non-negative

Leveraging auxiliary information

For example,

$$\underline{P}_t = \underline{P} + \sum_{r\in R} \underline{\alpha}_r Z_{r,t} \leq \overline{P} + \sum_{r\in R} \overline{\alpha}_r Z_{r,t} = \overline{P}_t, \quad t \in \mathcal{T}, \text{for all } Z_{r,t}$$

Assume that $Z_{r,t} \in [\overline{Z}_r, \underline{Z}_r]$, then

$$\underline{P} - \overline{P} + \underset{\substack{Z'_{r,t} \\ \text{s.t. } \underline{Z}_r \leq Z'_{r,t} \leq \overline{Z}_r, \quad r\in R}}{\text{Maximize}} \left\{ \sum_{r\in R} (\underline{\alpha}_r - \overline{\alpha}_r) Z'_{r,t} \right\} \leq 0, \quad t \in \mathcal{T}.$$

which is equivalent to

$$\begin{aligned}
&\overline{P} - \underline{P} + \sum_{r\in R} (\overline{\phi}_{r,t} \overline{Z}_r - \underline{\phi}_{r,t} \underline{Z}_r) \leq 0 && t \in \mathcal{T} \\
&\overline{\phi}_{r,t} - \underline{\phi}_{r,t} = \overline{\alpha}_r - \underline{\alpha}_r && r \in R, t \in \mathcal{T} \\
&\overline{\phi}_{r,t}, \underline{\phi}_{r,t} \geq 0 && r \in R, t \in \mathcal{T}.
\end{aligned}$$

Solving the estimation problem

- The estimation problem is **non-linear and non-convex**.
- We statistically **approximate its solution** by solving two **linear programming problems** instead.

  1. A **feasibility** problem (estimation of power bounds).
  2. An **optimality** problem (estimation of marginal utilities).

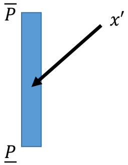
- A **two-step data driven estimation procedure** to achieve **optimality** and **feasibility** of x' in a statistical sense.

Feasibility problem: Estimation of power bounds

$$\underset{\underline{\boldsymbol{P}},\overline{\boldsymbol{P}},\boldsymbol{\xi},\boldsymbol{\mu},\boldsymbol{\alpha}}{\text{Minimize}} \sum_{t=1}^{T} \Bigg((1-\mathrm{K}) \left(\overline{\xi}_t^{+} + \underline{\xi}_t^{+} \right) + \mathrm{K} \left(\overline{\xi}_t^{-} + \underline{\xi}_t^{-} \right) \Bigg)$$

subject to

$$\overline{P}_t - x'_t = \overline{\xi}_t^{+} - \overline{\xi}_t^{-} \qquad \forall t$$
$$x'_t - \underline{P}_t = \underline{\xi}_t^{+} - \underline{\xi}_t^{-} \qquad \forall t$$
$$\underline{P}_t \leq \overline{P}_t \qquad \forall t$$
$$\underline{P}_t = \underline{\mu} + \sum_{r=1}^{R} \underline{\alpha}_r Z_{r,t} \qquad \forall t$$
$$\overline{P}_t = \overline{\mu} + \sum_{r=1}^{R} \overline{\alpha}_r Z_{r,t} \qquad \forall t$$
$$0 \leq \overline{\xi}_t^{+}, \overline{\xi}_t^{-}, \underline{\xi}_t^{+}, \underline{\xi}_t^{-} \qquad \forall t$$

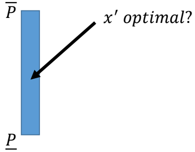


$\widehat{\overline{P}}_t, \underline{\widehat{P}}_t, \underline{\widehat{\mu}}, \widehat{\overline{\mu}}, \underline{\widehat{\alpha}}_r, \widehat{\overline{\alpha}}_r$

Optimality problem: Estimating marginal utilities

$$
\begin{aligned}
&\underset{\boldsymbol{\Omega}}{\text{Minimize}} \sum_{t=1}^{T} \epsilon_t \\
&\text{subject to } \widehat{\overline{P}}_t \overline{\lambda}_t - \widehat{\underline{P}}_t \underline{\lambda}_t + \sum_{b=1}^{B} E_b \overline{\phi}_{b,t} - \epsilon_t = \\
&\qquad \sum_{b=1}^{B} \tilde{x}'_{b,t}\left(u_{b,t} - p_t\right) && \forall t \\
&\qquad -\underline{\phi}_{b,t} + \overline{\phi}_{b,t} - \underline{\lambda}_t + \overline{\lambda}_t = u_{b,t} - p_t && \forall b, t \\
&\qquad u_{b,t} = \mu_b^u + \sum_{r} \alpha_r^u Z_{r,t} && \forall b, t \\
&\qquad \mu_b^u \geq \mu_{b+1}^u && \forall b < B \\
&\qquad \mu_1^u \geq 200 + \mu_2^u \\
&\qquad 0 \leq \overline{\lambda}_t, \underline{\lambda}_t, \underline{\phi}_{b,t}, \overline{\phi}_{b,t} && \forall b, t.
\end{aligned}
$$

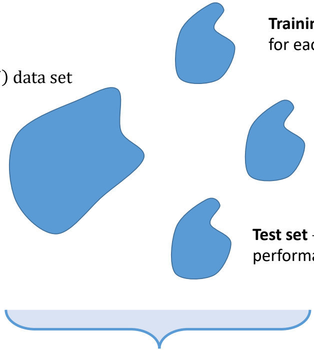
$\hat{u}_{b,t}, \hat{\mu}_b^u, \hat{\alpha}_r^u$



Solving the estimation problem

In the **bound estimation problem**, the **penalty parameter K** is statistically tuned through **validation**:

(x, p, Z) data set



Training set $\rightarrow$ Used for parametter fitting for each posible value of K

Validation set $\rightarrow$ Used to tune parameter K

Test set $\rightarrow$ Used to assess forecasting performance

We choose K so that the **out-of-simple prediction error** is minimized

K as indicator of the **price-responsiveness of the load**:

K

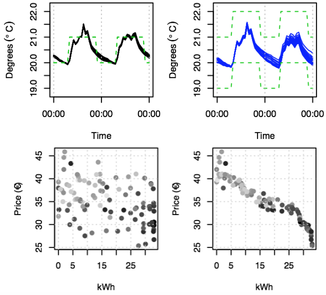
0 **Narrow** interval → **Small variability** of the load explained by the price.

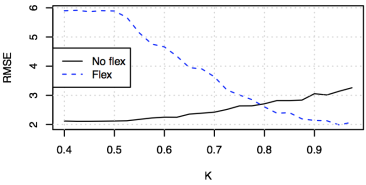
1 **Wide** interval → **High variability** of the load explained by the price.

Case study 1: One-hour ahead prediction

We simulate the price response behavior of a pool of **100 buildings** equipped with **heat pumps** (assuming economic MPC is in place).

Two classes of buildings are considered, depending on the **comfort bands of the indoor temperature**.





Case study 1: One-hour ahead prediction

We conduct a benchmark of the methodology against **simple persistence** forecasting and **autoregressive moving average with exogenous inputs**.

- **Simple persistence model**: The forecast load at time t is set to be equal to the observed load at $t-1$.
- **ARMAX**: The aggregate load x is a linear combination of the past values of the load, past errors and regressors.

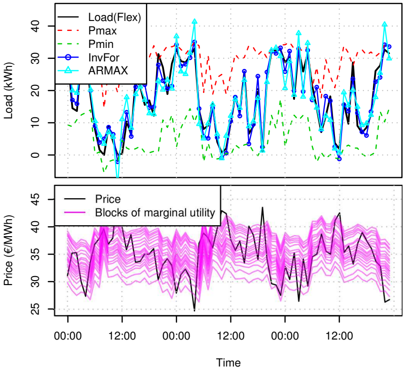
$$x_t = \mu + \epsilon_t + \sum_{p=1}^{P} \varphi_p x_{t-p} + \sum_{r=1}^{R} \gamma_r Z_{t-r} + \sum_{q=1}^{Q} \theta_q \epsilon_{t-q}$$

Forecasting performance is evaluated according to MAE and

$$NRMSE = \frac{1}{x^{max} - x^{min}} \sqrt{\frac{1}{T} \sum_{t=1}^{T} \left(\sum_{b=1}^{B} \widehat{x}_{b,t} - x'_t \right)^2}$$

$$\mathrm{MASE} = \frac{\sum_{t=1}^{T} \left| \sum_{b=1}^{B} \widehat{x}_{b,t} - x'_t \right|}{\frac{T}{T-1} \sum_{t=2}^{T} \left| x'_t - x'_{t-1} \right|}$$

Case study 1: One-hour ahead prediction



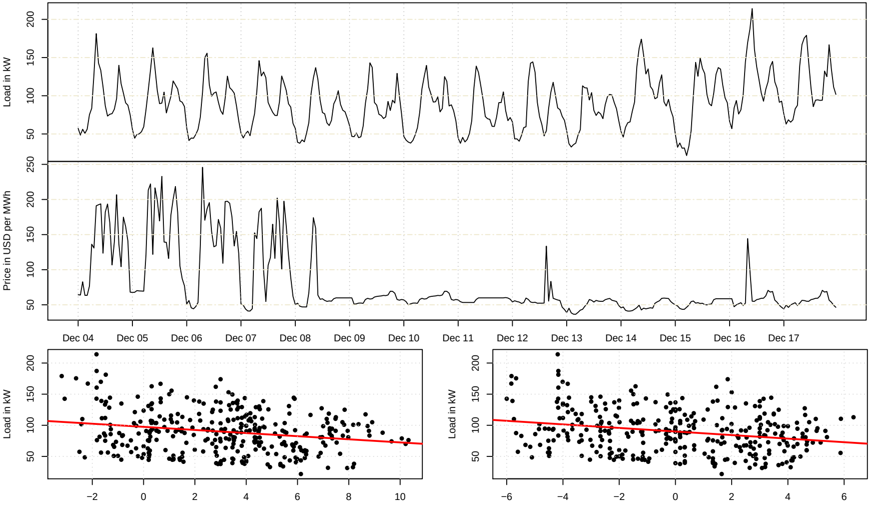
		MAE	NRMSE	MASE
No Flex	*Persistence*	4.092	0.155	-
	ARMAX	2.366	0.097	0.578
	InvFor	2.275	0.096	0.556
Flex	*Persistence*	8.366	0.326	-
	ARMAX	2.948	0.112	0.352
	InvFor	2.369	0.097	0.283

Case study 2: One-day (24-h) ahead prediction

- Data of price-responsive households from Olympic Peninsula project from May 2006 to March 2007
- Decisions made by the home-automation system based on occupancy modes, comfort settings, and price
- The price was sent out every 15 minutes to 27 households

Case study 2: One-day (24-h) ahead prediction

- Load, price, temperature and dew point during December



Benchmark models

ARX: Auto-Regressive model with eXogenous inputs [Dorini et al., 2013, Corradi et al., 2013]

$$x_t = \vartheta_x \boldsymbol{X}_{t-n} + \vartheta_z \boldsymbol{Z}_t + \epsilon_t,$$

with $\epsilon_t \sim \mathrm{N}(0,\sigma^2)$ and σ^2 is the variance.

$\boldsymbol{Z}_t$: outside temperature, solar irradiance, wind speed, humidity, dew point (up to 36 hours in the past), plus binary indicators for the hour of the day and the day of the week.

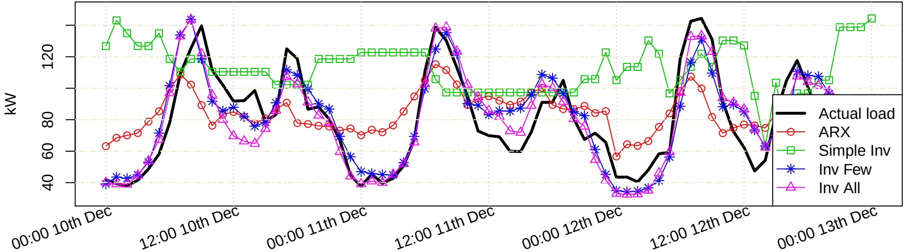
Simple Inv: Only the marginal utilities are estimated (12 blocks) as in Step 2, the rest of bid parameters to historical maximum/minimum values observed in the last seven days. Inspired from Keshavarz et al. [2011], Chan et al. [2014].

Inv Few: Our inverse optimization scheme only with the outside temperature and hourly indicator variables as features.

Inv All: The same as **Inv Few**, but including all features.

Case study 2: One-day (24-h) ahead prediction

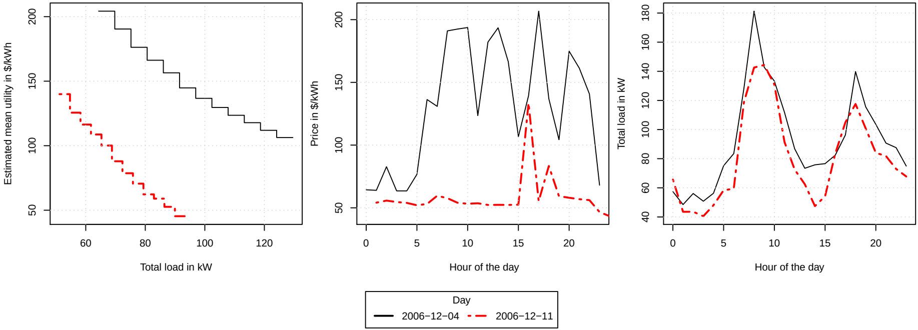
Prediction capabilities of different benchmarked methods



	MAE	RMSE	MAPE
ARX	22.17692	27.50130	0.2752790
Simple Inv	44.43761	54.57645	0.5858138
Inv Few	16.92597	22.27025	0.1846772
Inv All	17.55378	22.39218	0.1987778

Case study 2: One-day (24-h) ahead prediction

Estimated marginal utility for the pool of price-responsive consumers

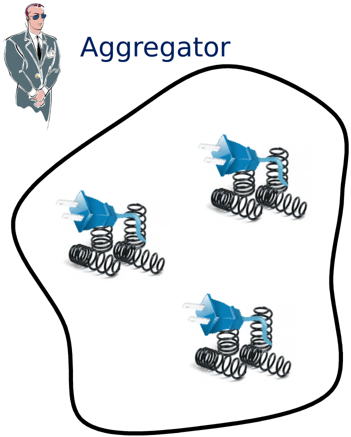


Case study 2: One-day (24-h) ahead prediction

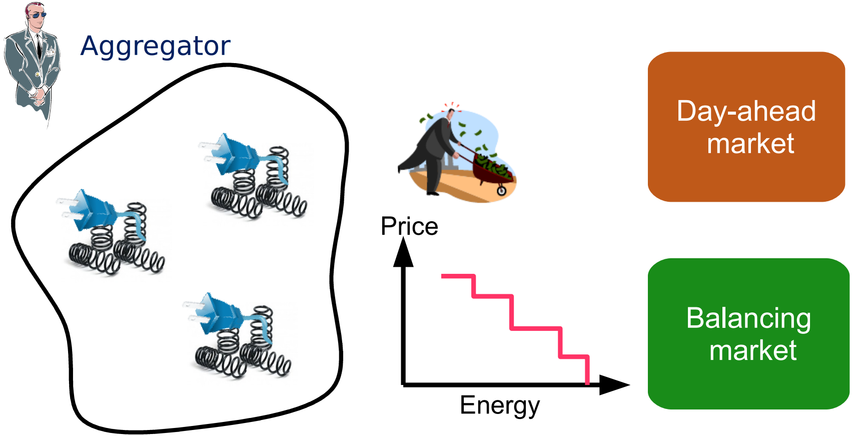
	September			March		
	MAE	RMSE	MAPE	MAE	RMSE	MAPE
ARX	7.6499	9.8293	0.2358	17.4397	23.3958	0.2602
Simple Inv	14.2631	17.8	0.4945	44.6872	54.6165	0.8365
Inv Few	5.5031	7.9884	0.1464	13.573	17.9454	0.2103
Inv All	5.8158	8.4941	0.1511	14.7977	19.1195	0.2391

The prediction performance of the proposed machinery is only slightly lower than that of the state-of-the-art prediction tool developed in Hosking et al. [2013] on the same dataset. However, our methodology produces a market bid!

Alternative application: Market bidding



Alternative application: Market bidding



Simple bid

$$\text{BID} = \{u_{b,t}, \forall t, \forall b; E_b, \forall b; \underline{P}_t, \overline{P}_t, \forall t\}$$

$$\begin{aligned} \underset{x_{b,t}, \forall b,t}{\text{Maximize}} \quad & \sum_{b=1}^{B} x_{b,t}(u_{b,t} - p_t) \\ \text{subject to} \quad & \underline{P}_t \leq \sum_{b=1}^{B} x_{b,t} \leq \overline{P}_t, \quad \forall t \\ & 0 \leq x_{b,t} \leq E_b, \quad \forall t \end{aligned}$$

Complex bid

$$\text{BID} = \{u_{b,t}, \forall t, \forall b; E_b, \forall b; \underline{P}_t, \overline{P}_t, r_t^u, r_t^d, \forall t\}$$

Total consumption: $\underline{P}_t + \sum_{b \in B} x_{b,t}$

$$\underset{x_{b,t}}{\text{Maximize}} \sum_{t \in \mathcal{T}} \left(\sum_{b \in B} u_{b,t} x_{b,t} - p_t \sum_{b \in \mathcal{B}} x_{b,t} \right)$$

subject to

$$\underline{P}_t + \sum_{b \in B} x_{b,t} - \underline{P}_{t-1} - \sum_{b \in B} x_{b,t-1} \le r_t^u \qquad t \in \mathcal{T}_{-1}$$

$$\underline{P}_{t-1} + \sum_{b \in B} x_{b,t-1} - \underline{P}_t - \sum_{b \in B} x_{b,t} \le r_t^d \qquad t \in \mathcal{T}_{-1}$$

$$0 \le x_{b,t} \le E_b \qquad b \in B, t \in \mathcal{T}$$

Conclusions

- A new method to forecast price-responsive electricity consumption **one-step/multiple-steps ahead**.
- The method can be exploited to produce **market bids** for flexible electricity consumers
- A **two-step algorithm** to statistically approximate the exact inverse-optimization solution.
- A **validation scheme** to minimize the out of sample prediction error.
- The proposed methodology has been evaluated on:
  - A synthetic data set corresponding to a cluster of price-responsive buildings equipped with a heat pump and MPC.
  - A data set from a **real-world experiment** involving electricity consumers able to react to the electricity price.
- The **non-linearity between price and aggregate load** is well described by our methodology.

Future Work

- Dealing with errors in the measurements and with bounded rationality (suboptimality).
- Examining more flexible functional forms between model parameters and regressors.
- Investigating statistically consistent set-valued functions (feasibility set as a function of regressors)
- Testing the methodology on other data sets.

Contacts

Any questions?

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Full papers
Short-term forecasting of price-responsive loads using inverse optimization and
A data-driven bidding model for a cluster of price-responsive consumers of electricity
are available online at IEEExplore
http://ieeexplore.ieee.org/document/7859377/
https://ieeexplore.ieee.org/document/7416249/

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