Rješenja drugog međuispita iz Matematike 3E
29.11.2007.

1. (4 boda)

$$\int_0^1 dx \int_{-1}^{-\sqrt{x}} f(x,y)\,dy + \int_0^1 dx \int_{\sqrt{x}}^{1} f(x,y)\,dy$$

$$\int_{-1}^{1} dy \int_0^{y^2} f(x,y)\,dx$$

2. (3 boda)

$$P = \iint_D dx\,dy = \iint_{D'} r\,dr\,d\varphi = \int_0^{2\pi} d\varphi \int_0^{1+\sin\varphi} r\,dr = \cdots = \frac{3\pi}{2}$$

3. (3 boda)

$$\iiint_V x\,dx\,dy\,dz = \int_0^1 x\,dx \int_0^{2-2x} dy \int_0^{\frac{y}{2}} dz = \cdots = \frac{1}{12}$$

4. (3 boda)

$$\int_0^{\frac{\pi}{2}} d\varphi \int_0^1 dr \int_r^1 g(\varphi,r,z) r\,dz$$

5. (5 bodova)

a) (**1b**)

$$\begin{aligned} x &= r\sin\theta\cos\varphi \\ y &= r\sin\theta\sin\varphi \\ z &= r\cos\theta \end{aligned}$$

b) (**1b**)

$$J = \cdots = r^2\sin\theta$$

c) (**3b**)

$$\begin{aligned} x &= r\sin\theta\cos\varphi \\ y &= r\sin\theta\sin\varphi \\ z &= r\cos\theta + 3 \end{aligned}$$

$$\iiint_V \left(x^2+y^2+z^2\right)\,dx\,dy\,dz = \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \int_0^3 \left(r^2 + 6r\cos\theta + 9\right) r^2\sin\theta\,dr = \cdots = \frac{2592\pi}{5}$$

6. (7 bodova)

a) (**1b**)

$$\mathbf{r}(u) = (x(t_0)\mathbf{i} + y(t_0)\mathbf{j} + z(t_0)\mathbf{k}) + \left(x'(t_0)\mathbf{i} + y'(t_0)\mathbf{j} + z'(t_0)\mathbf{k}\right)\cdot u, \quad u \in \mathbb{R}$$

b) (**1b**)

$$\mathbf{r}'(t) = (-2\sin t)\mathbf{i} + (3\cos t)\mathbf{j} + \mathbf{k}$$

$$\mathbf{r}(\pi) = \cdots = -2\mathbf{i} + \pi\mathbf{k}$$

$$\mathbf{r}'(\pi) = \cdots = -3\mathbf{j} + \mathbf{k}$$

$$\mathbf{r}(u) = (-2\mathbf{i} + \pi\mathbf{k}) + (-3\mathbf{j} + \mathbf{k}) \cdot u, \quad u \in \mathbb{R}$$

c) (**3b**)

$$z = x^2 + y^2, \quad 2x + z = 0 \quad \Rightarrow \quad z = -2x$$

$$x^2 + y^2 = -2x \quad \Rightarrow \quad x^2 + 2x + 1 + y^2 = 1 \quad \Rightarrow \quad (x+1)^2 + y^2 = 1$$

$$x = -1 + \cos t, \quad y = \sin t \quad \Rightarrow \quad z = 2 - 2\cos t, \qquad t \in [0, 2\pi]$$

d) (**2b**)

$$\mathbf{r}'(t) = (-2\sin t)\mathbf{i} + (2\cos t)\mathbf{j} - 2t\mathbf{k}$$

$$\sqrt{5} = \left|r'(t)\right| = \sqrt{(-2\sin t)^2 + (2\cos t)^2 + (-2t)^2} \quad \Rightarrow \quad \cdots \quad \Rightarrow \quad t_{1,2} = \pm\frac{1}{2}$$