Rješenja ponovljenog završnog ispita iz Matematike 3E
04.02.2010.

Pitanja iz 3. ciklusa nastave (25 bodova)

1. (5 bodova)
a) (**2b**)

$$\frac{\partial \mathbf{v}}{\partial \mathbf{s}} = \frac{yz+xy}{\sqrt{2}}\mathbf{i} + \frac{2x}{\sqrt{2}}\mathbf{j} + \frac{y^2}{\sqrt{2}}\mathbf{k}.$$

b) (**1b**)
Raspisati po koordinatama ili koristiti formalni račun sa nabla operatorom.
c) (**2b**)

$$\operatorname{div}\Big(\frac{\mathbf{a}\times\mathbf{r}}{\mathbf{a}\cdot\mathbf{r}}\Big) = 0.$$

2. (4 boda)

$$\int_\Gamma ds = \frac{8\pi}{3}.$$

3. (3 boda)
Ako postoji $f(z)$, mora vrijediti $\operatorname{rot}\mathbf{v} = \mathbf{0} \quad \Rightarrow \quad f'(z) = 3z^2 - 1 \quad \Rightarrow \quad f(z) = z^3 - z + C, \quad C \in \mathbb{R}$.

4. (3 boda)
a) (**1b**) Skripta, str. 54, Teorem 1.
b) (**2b**)

$$\oint_\Gamma y\,dx + x\,dy = \iint_\Omega \left(\frac{\partial x}{\partial x} - \frac{\partial y}{\partial y}\right) dxdy = 0.$$

5. (5 bodova)

$$\oint_\Gamma x^4\,dx + 2y\,dy + xyz\,dz = \oint_\Gamma \mathbf{a}\cdot d\mathbf{r} = \iint_S (\operatorname{rot}\mathbf{a})\cdot\mathbf{n}\,dS = \iint_S xz\,dydz - yz\,dxdz = \pi\sqrt{2}.$$

6. (5 bodova)

$$\iint_S xy\,dxdy + y\,dxdz + x\,dydz = \frac{16\pi}{3}.$$

Pitanja iz cijelog gradiva (15 bodova)

7. (4 boda)

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\sum_{k=0}^{\infty} \frac{\cos((2k+1)x)}{(2k+1)^2}.$$

8. (4 boda)

a) (**1b**) Skripta, str. 69, 3.1.

b) (**3b**)

$$\mathscr{L}\left(e^{-2t}\right) = F(s) = \int_0^\infty e^{-st} \cdot e^{-2t}\, dt = \frac{1}{s+2},$$

$$\mathscr{L}\left(e^{-2t}\right) \text{ postoji za } s > -2.$$

9. (4 boda)

a) (**1b**) Skripta, str. 66, Definicija 1.

b) (**3b**)

$$\iint_S (x^2 + y^2)\, dS = \frac{4\pi}{3}.$$

10. (3 boda)

$$\frac{x-1}{0} = \frac{y-0}{2} = \frac{z-4\pi}{1} = u, \quad u \in \mathbb{R}.$$