

Rješenja završnog ispita iz Matematike 3E
27.01.2010.

Pitanja iz 3. ciklusa nastave (25 bodova)

1. (6 bodova)

a) (1b)

$$\frac{\partial \mathbf{r}}{\partial \mathbf{s}} = \frac{\mathbf{i} + \mathbf{j} + \mathbf{k}}{\sqrt{3}}.$$

b) (3b)

$$\Delta r = \frac{2}{r}.$$

c) (2b) Lagani račun po definiciji rot.

2. (7 bodova)

a) (2b) Skripta, str. 56, Teorem 2, dovoljnost.

b) (1b)

$$\operatorname{rot} \mathbf{v} = \operatorname{rot}(\operatorname{grad} p) = \dots = \mathbf{0}.$$

c) (3b)

$$\operatorname{rot} \mathbf{v} = \dots = \mathbf{0} \quad \Rightarrow \quad \mathbf{v} \text{ je potencijalno.}$$

$$\text{Potencijal } p(x, y, z) = ye^{xy+z} + C.$$

d) (1b)

$$\int_{(0,1,\pi/2)}^{(1,0,2\pi)} \mathbf{v} \cdot d\mathbf{r} = p(1, 0, 2\pi) - p(0, 1, \pi/2) = -e^{\pi/2}.$$

3. (3 boda)

$$\int_{\Gamma} y ds = \frac{4}{3}(3\sqrt{3} - 1).$$

4. (4 boda)

$$\iint_S (\operatorname{rot} \mathbf{a}) \cdot \mathbf{n} dS = \oint_{\mathcal{K}} \mathbf{a} \cdot d\mathbf{r} = 27\pi.$$

5. (5 bodova)

$$\iint_S x dydz + y dxdy = -\pi\sqrt{2}.$$

Pitanja iz cijelog gradiva (15 bodova)

6. (4 boda)

Funkcija je neparna $\Rightarrow a_n = 0, \quad b_n = -\frac{2}{n\pi}((-1)^n - 1).$

$$f(x) = \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{\sin((2n+1)x)}{2n+1}.$$

7. (3 boda)

$$f(t) = \sin t \cdot g_{[0,\pi]}(t) = \sin t(u(t) - u(t-\pi)) = \sin t \cdot u(t) + \sin(t-\pi)u(t-\pi) \quad \circ \bullet \quad \frac{1}{s^2+1} + \frac{e^{-\pi s}}{s^2+1}.$$

8. (4 boda)

a) **(1b)**

$$x = r \cos \varphi, \quad y = r \sin \varphi, \quad z = z, \quad dx dy dz = r dr d\varphi dz.$$

$$r \in [0, +\infty), \quad \varphi \in [0, 2\pi), \quad z \in \langle -\infty, +\infty \rangle.$$

b) **(3b)**

$$\int_0^2 dx \int_0^{\sqrt{2x-x^2}} dy \int_0^3 dz = \int_0^{\pi/2} d\varphi \int_0^{2 \cos \varphi} r dr \int_0^3 dz = \frac{3}{2}\pi.$$

9. (3 boda)

$$\iint_S dS = \iint_{\Omega_{xy}} \sqrt{2} dx dy = 3\sqrt{2}\pi.$$

10. (1 bod)

Skripta, str. 71, Teorem 1.