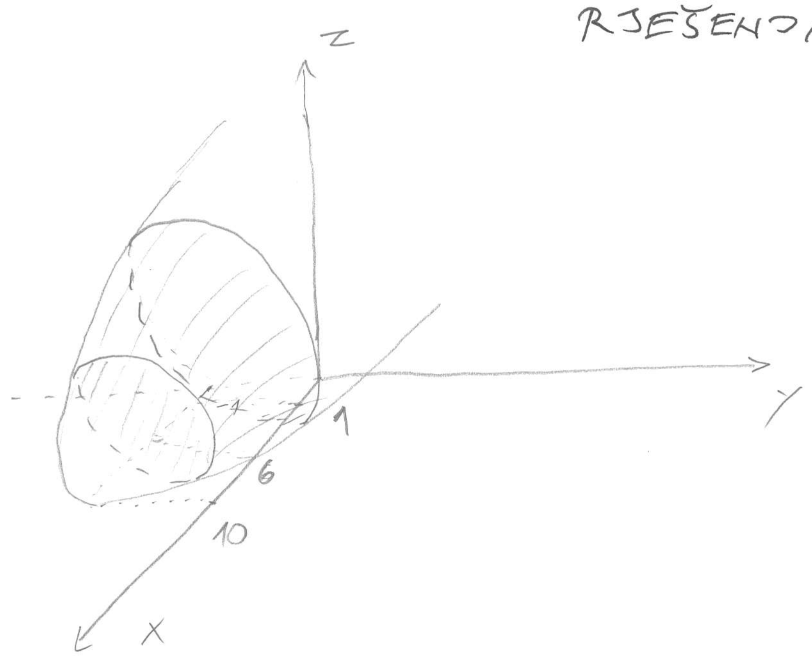
3. SKZ. V2. A

RJEŠENJA

1.)



POMAKNUTE CILINDRIČNE KOORDINATE: $\begin{cases} y = -1 + r\cos\varphi \\ z = r\sin\varphi \\ x = x \end{cases}$, $|J| = r$

$$I = \int_0^{2\pi} d\varphi \int_1^6 \frac{dx}{x^2} \int_0^{\sqrt{10-x}} r\,dr$$

$$= \ldots = \pi \int_1^6 \frac{10-x}{x^2}\,dx = \ldots = \frac{25\pi}{3} - \pi \ln 6$$

2.) Rj

$$\sqrt{x^2+y^2} = 2 - \sqrt{x^2+(y-1)^2}$$

$$\sqrt{x^2+(y-1)^2} = 2 - \sqrt{x^2+y^2} \quad /^2$$

$$\ldots \quad 4\sqrt{x^2+y^2} = 3+2y \quad /^2$$

$$\ldots \quad \left(\frac{x}{\frac{\sqrt{3}}{2}}\right)^2 + \left(y - \frac{1}{2}\right)^2 = 1$$

$$\Rightarrow \begin{cases} x = \frac{\sqrt{3}}{2}\cos t \\ y = \frac{1}{2} + \sin t, \quad t \in [0, 2\pi] \end{cases} \Rightarrow \boxed{t_0 = 0} \qquad T\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$$

$$z = \sqrt{x^2+y^2} = \sqrt{\frac{3}{4}\cos^2 t + \sin^2 t + \sin t + \frac{1}{4}}$$

$$\dot{x}(t) = -\frac{\sqrt{3}}{2}\sin t\Big|_{t=0} = 0$$

$$\dot{y}(t) = \cos t\Big|_{t=0} = 1$$

$$\dot{z}(t) = \frac{\sin t\cos t + 2\cos t}{2\sqrt{3\cos^2 t + 4\sin^2 t + 4\sin t + 1}}\Bigg|_{t=0} = \frac{1}{2}$$

$$\Rightarrow \quad t\ldots \quad \frac{x - \frac{\sqrt{3}}{2}}{0} = \frac{y - \frac{1}{2}}{1} = \frac{z-1}{\frac{1}{2}}$$

3.) $\nabla f(x,y,z) = 2x\vec{i} + (y+3)\vec{j} + 4z\vec{k}$

$$\overset{\text{UVJET}}{=} \lambda(\vec{i} + 2\vec{j} + 3\vec{k}), \quad \lambda \neq 0$$

$$\Leftrightarrow \quad (*)\begin{cases} x = \frac{\lambda}{2} \\ y = 2\lambda - 3 \\ z = \frac{3}{4}\lambda \end{cases}$$

T JE NA PLOHI $\Rightarrow$

$$\frac{1}{4}\lambda^2 + \frac{1}{2}(2\lambda)^2 + 2\left(\frac{3}{4}\lambda\right)^2 = 1$$

$$\ldots \quad \lambda_{1,2} = \pm\sqrt{\frac{8}{27}} = \pm 2\sqrt{\frac{2}{27}}$$

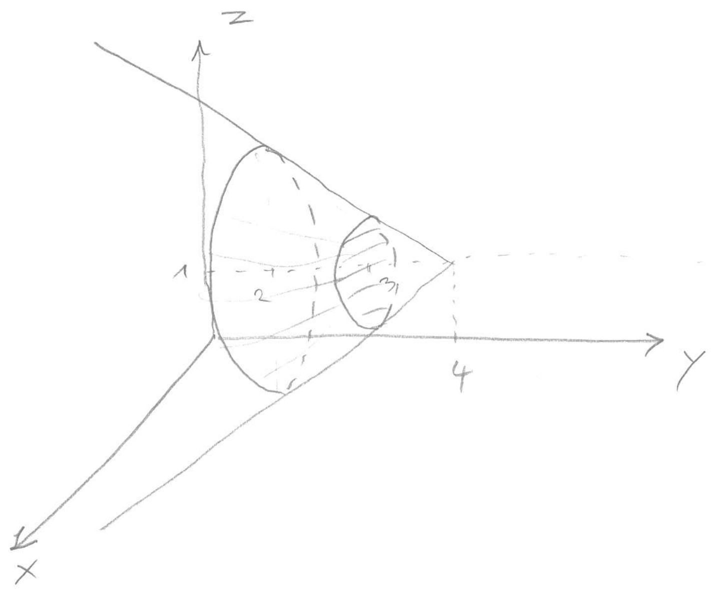
$$\overset{(*)}{\Rightarrow} \quad T_1\left(\sqrt{\frac{2}{27}},\ 4\sqrt{\frac{2}{27}} - 3,\ \frac{3}{2}\sqrt{\frac{2}{27}}\right)$$

$$T_2\left(-\sqrt{\frac{2}{27}},\ -4\sqrt{\frac{2}{27}} - 3,\ -\frac{3}{2}\sqrt{\frac{2}{27}}\right)$$

3. SKZ. V2. B.

RJEŠENJA

1.)



POMAKNUTE CILINDRIČNE KOORDINATE: $\begin{cases} z = 1 + r\cos\varphi \\ x = r\sin\varphi \\ y = y \end{cases}$, $|J| = r$

$$I = \int_0^{2\pi} d\varphi \int_2^3 \frac{dy}{y} \int_0^{4-y} r\,dr = \ldots = \underline{\underline{16\pi \ln\frac{3}{2} - \frac{11\pi}{2}}}$$

2.) $$3x^2 + y^2 = -(x-1)^2 - 3y^2 + 1$$

$$\vdots$$

$$\left(\frac{x - \frac{1}{4}}{\frac{1}{4}}\right)^2 + \left(\frac{y}{\frac{1}{4}}\right)^2 = 1$$

$$\Rightarrow \begin{cases} x = \frac{1}{4} + \frac{1}{4}\cos t \\ y = \frac{1}{4}\sin t \end{cases}, \quad t \in [0, 2\pi]$$

$$z = 3x^2 + y^2 = \frac{1}{16}\sin^2 t + \frac{3}{16}\cos^2 t + \frac{3}{8}\cos t + \frac{3}{16}$$

$$T\left(\frac{1}{4}, -\frac{1}{4}, \frac{1}{4}\right) \Rightarrow \boxed{t_0 = \frac{3\pi}{2}}$$

$$\dot{x}(t) = -\frac{1}{4}\sin t \Big|_{t=\frac{3\pi}{2}} = \frac{1}{4}$$

$$\dot{y}(t) = \frac{1}{4}\cos t \Big|_{t=\frac{3\pi}{2}} = 0$$

$$\dot{z}(t) = -\frac{1}{4}\sin t\cos t - \frac{3}{8}\sin t \Big|_{t=\frac{3\pi}{2}} = \frac{3}{8}$$

$$\Rightarrow \quad t\ldots \quad \underline{\underline{\frac{x-\frac{1}{4}}{\frac{1}{4}} = \frac{y+\frac{1}{4}}{0} = \frac{z-\frac{1}{4}}{\frac{3}{8}}}}$$

3.) $\nabla f(x,y,z) = 8x\vec{i} + 2y\vec{j} - \frac{1}{2}(z+2)\vec{k}$

$$\overset{\text{UVJET}}{=} \lambda\left(\sqrt{3}\,\vec{i} + \frac{1}{\sqrt{3}}\vec{j} + \frac{1}{\sqrt{2}}\vec{k}\right), \quad \lambda \neq 0$$

$$\Leftrightarrow \left\{\begin{aligned} 8x &= \lambda\sqrt{3} \\ 2y &= \frac{\lambda}{\sqrt{3}} \\ -\frac{1}{2}(z+2) &= \frac{\lambda}{\sqrt{2}} \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} x &= \frac{\sqrt{3}}{8}\lambda \\ y &= \frac{\lambda}{2\sqrt{3}} \\ z &= -2 - \frac{2\lambda}{\sqrt{2}} \end{aligned}\right\} (*)$$

T JE NA PLOHI $\Rightarrow$

$$4\cdot\frac{3}{64}\lambda^2 + \frac{\lambda^2}{4\cdot 3} - \frac{1}{4}\cdot\frac{4\lambda^2}{2} = -1$$

$$\ldots \quad \underline{\lambda_{1,2} = \pm 4\sqrt{\frac{3}{11}}}$$

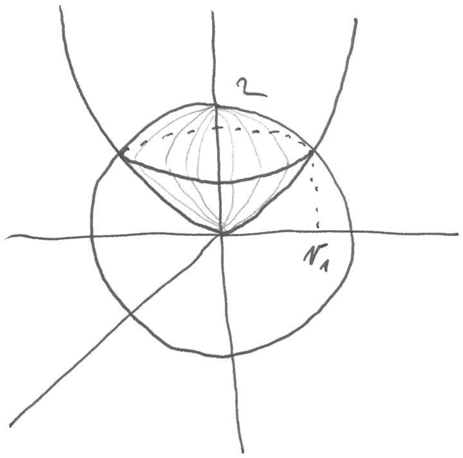
$$(*) \Rightarrow \left\{\begin{aligned} &T_1\left(\frac{3}{2\sqrt{11}}, \frac{2}{\sqrt{11}}, -2 - \frac{8\sqrt{3}}{\sqrt{22}}\right) \\ &T_2\left(-\frac{3}{2\sqrt{11}}, -\frac{2}{\sqrt{11}}, -2 + \frac{8\sqrt{3}}{\sqrt{22}}\right) \end{aligned}\right.$$

(1)
V1.A

3. SKZ. V1. A

RJEŠENJA

1.)



CILINDRIČNE KOORDINATE : $\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \\ z = z \end{cases}$ $\quad |J| = r$

ODREDITI r_1 (NPR. PROJEKCIJA NA YZ RAVNINU):

$$\left\{\begin{array}{l} y^2 + z^2 = 4 \\ y^2 = 3z \end{array}\right\} \Rightarrow 3z + z^2 = 4 \Rightarrow \ldots \Rightarrow z_1 = -2 \quad \lightning$$

$$\boxed{z_2 = 1} \checkmark$$

$$r_1^2 = y^2 = 3z_2 \Rightarrow \boxed{r_1 = \sqrt{3}}$$

$$V = \int_0^{2\pi} d\varphi \int_0^{\sqrt{3}} r\,dr \int_{r^2/3}^{\sqrt{4-r^2}} dz = \ldots = \frac{19\pi}{6}$$

2.)

$$\begin{cases} x^2 + y^2 + z^2 = 25 \\ x^2 - y^2 = 16 \end{cases} \qquad T(-4, 0, -3)$$

$$x^2 - y^2 = 16 \Rightarrow x^2 = 16 + y^2$$

$$16 + 2y^2 + z^2 = 25$$

$$\ldots \quad \frac{y^2}{\frac{9}{2}} + \frac{z^2}{9} = 1 \Rightarrow \boxed{\begin{cases} y = \frac{3}{\sqrt{2}}\cos t \\ z = 3\sin t \end{cases}, \quad t \in [0, 2\pi]}$$

$$x = \pm\sqrt{16 + \frac{9}{2}\cos^2 t}, \quad x_0 = -4 < 0 \Rightarrow \boxed{x = -\sqrt{16 + \frac{9}{2}\cos^2 t}}$$

$$3\sin t = z_0 = -3 \Rightarrow \boxed{t_0 = \frac{3\pi}{2}}$$

$$x'(t) = \frac{9\cos t \sin t}{2\sqrt{16 + \frac{9}{2}\cos^2 t}}\Bigg|_{t=\frac{3\pi}{2}} = 0$$

$$y'(t) = -\frac{3}{\sqrt{2}}\sin t\Bigg|_{t=\frac{3\pi}{2}} = \frac{3}{\sqrt{2}}$$

$$z'(t) = 3\cos t\Bigg|_{t=\frac{3\pi}{2}} = 0$$

$$\Rightarrow \quad t \ldots \quad \frac{x+4}{0} = \frac{y}{\frac{3}{\sqrt{2}}} = \frac{z+3}{0}$$

3.)

$$\vec{s}_0 = \frac{1}{\sqrt{3}}(\vec{i} - \vec{j} + \vec{k})$$

$$(\vec{s}_0 \cdot \nabla)\vec{a} = \frac{1}{\sqrt{3}}\left(\frac{\partial}{\partial x} - \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\left[(x^2 - yz)\vec{i} + xz\vec{j} + z^2\vec{k}\right]$$

$$= \frac{1}{\sqrt{3}}\left[(2x + z - y)\vec{i} + (z + x)\vec{j} + 2z\vec{k}\right]$$

UVJET

$$= \lambda(3\vec{i} + 2\vec{j} + \vec{k}), \quad \lambda \neq 0$$

$$\Leftrightarrow \begin{cases} 2x + z - y = 3\tilde{\lambda} \\ z + x = 2\tilde{\lambda} \\ 2z = \tilde{\lambda} \end{cases}, \quad \tilde{\lambda} \neq 0 \quad (\tilde{\lambda} = \lambda\sqrt{3})$$

$$\boxed{t := z \in \mathbb{R}}$$

$$\Rightarrow \quad t + x = 2 \cdot 2t \Rightarrow \boxed{x = 3t}$$

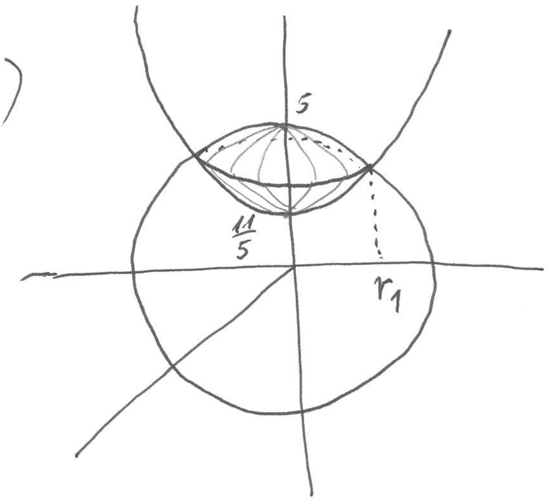
$$\boxed{y = 6t + t - 3 \cdot 2t = t}$$

$$\Rightarrow \quad \text{PRAVAC} \quad p \ldots \quad \frac{x}{3} = \frac{y}{1} = \frac{z}{1}$$

3. SKZ. V1. B

RJEŠENJA

1.)



CILINDRIČNE KOORDINATE : $\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \\ z = z \end{cases}$, $|J| = r$

ODREDITI r_1 (NPR. PROJEKCIJA NA yz RAVNINU):

$$\left\{\begin{matrix} y^2+z^2=25 \\ y^2 = 5z-11 \end{matrix}\right\} \Rightarrow z^2+5z-36=0 \Rightarrow \dots \Rightarrow z_1=-9 \ ↯$$

$$\boxed{z_2 = 4} \checkmark$$

$$r_1^2 = y^2 = 5z_2 - 11 = 9 \Rightarrow \boxed{r_1 = 3}$$

$$V = \int_0^{2\pi} d\varphi \int_0^3 r\,dr \int_{\frac{r^2}{5}+\frac{11}{5}}^{\sqrt{25-r^2}} dz = \dots = \underline{\underline{\frac{383\pi}{30}}}$$

2.) $\begin{cases} z = \frac{x^2}{4} + y^2 + 1 \\ z = x^2 - y^2 \end{cases}$ $T(-2,-1,3)$

$$\frac{x^2}{4} + y^2 + 1 = x^2 - y^2$$

$$\dots \quad x = \pm \frac{2}{\sqrt{3}}\sqrt{2y^2+1} \quad , \quad x_0 = -2 < 0 \Rightarrow$$

$$\boxed{x = -\frac{2}{\sqrt{3}}\sqrt{2y^2+1}} \quad , \quad \boxed{y =: t \in \mathbb{R}}$$

$$z = x^2 - y^2 = \boxed{\frac{5t^2}{3} + \frac{4}{3}}$$

$$y(t_0) = -1 \Rightarrow \boxed{t_0 = -1}$$

$$x'(t) = -\frac{4t}{\sqrt{3}\sqrt{2t^2+1}}\Bigg|_{t=-1} = \frac{4}{3}$$

$$y'(t_0) = 1$$

$$z'(t) = \frac{10}{3}t\Bigg|_{t=-1} = -\frac{10}{3}$$

$$\Rightarrow t \ldots \quad \frac{x+2}{\frac{4}{3}} = \frac{y+1}{1} = \frac{z-3}{-\frac{10}{3}}$$

3.) $$\vec{s}_0 = \frac{\vec{i}-\vec{k}}{\sqrt{2}}$$

$$\frac{\partial f(r)}{\partial \vec{s}} = (\vec{s}_0 \cdot \nabla) f(r) = \vec{s}_0 \cdot (\nabla f(r))$$

$$= \vec{s}_0 \cdot f'(r)\nabla r = f'(r)(\vec{s}_0 \cdot \vec{r}_0)$$

$$\vec{r}_0 = \frac{\vec{r}}{r}, \quad f'(r) = -\frac{2}{r^3}$$

$$\Rightarrow \frac{\partial f(r)}{\partial \vec{s}} = -\frac{2}{r^3}\,\frac{\vec{i}-\vec{k}}{\sqrt{2}} \cdot \frac{x\vec{i}+y\vec{j}+z\vec{k}}{r}$$

$$= -\frac{2}{\sqrt{2}}\,\frac{x-z}{(x^2+y^2+z^2)^2} \overset{\text{UVJET}}{=} 0 \Leftrightarrow x = z$$

$$\Rightarrow \left\{\begin{array}{l} x = z \\ x^2+y^2+z^2 = 1 \end{array}\right\} \Rightarrow \begin{array}{l} 2x^2+y^2 = 1 \\ \left(\frac{x}{\frac{1}{\sqrt{2}}}\right)^2 + y^2 = 1 \end{array}$$

$$\Rightarrow \left\{\begin{array}{l} x = \frac{1}{\sqrt{2}}\cos t \\ y = \sin t \\ z = \frac{1}{\sqrt{2}}\cos t \end{array}\right. , \quad t \in [0, 2\pi]$$