

**Osnovne trigonometrijske jednakosti**

$$\sin\left(x \pm \frac{\pi}{2}\right) = \pm \cos x$$

$$\cos\left(x \pm \frac{\pi}{2}\right) = \mp \sin x$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\sin x - \sin y = 2 \sin \frac{x-y}{2} \cos \frac{x+y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\cos x - \cos y = 2 \sin \frac{x+y}{2} \sin \frac{y-x}{2}$$

$$\sin x \sin y = \frac{1}{2} (\cos(x-y) - \cos(x+y))$$

$$\cos x \cos y = \frac{1}{2} (\cos(x-y) + \cos(x+y))$$

$$\sin x \cos y = \frac{1}{2} (\sin(x-y) + \sin(x+y))$$

$$\sin(2x) = 2 \sin x \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$2 \sin^2 x = 1 - \cos(2x)$$

$$2 \cos^2 x = 1 + \cos(2x)$$

**Tablice suma i integrala****Konačne sume**

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{i=0}^n x^i = \frac{x^{n+1} - 1}{x - 1}$$

$$\sum_{i=0}^n e^{j(\theta+i\phi)} = \frac{\sin((n+1)\phi/2)}{\sin(\phi/2)} e^{j(\theta+n\phi/2)}$$

$$\sum_{i=0}^n \binom{n}{i} = \sum_{i=1}^n \frac{n!}{i!(n-i)!} = 2^n$$

**Neodređeni integrali****Racionalne funkcije**

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)}, \quad 0 < n$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln |ax+b|$$

$$\int \frac{dx}{a^2x^2+b^2} = \frac{1}{ab} \operatorname{tg}^{-1}\left(\frac{ax}{b}\right)$$

$$\int \frac{x dx}{a^2+x^2} = \frac{1}{2} \ln(a^2+x^2)$$

$$\int \frac{x^2 dx}{a^2+x^2} = x - a \operatorname{tg}^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{dx}{(a^2+x^2)^2} = \frac{x}{2a^2(a^2+x^2)} + \frac{1}{2a^3} \operatorname{tg}^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{x dx}{(a^2+x^2)^2} = \frac{-1}{2(a^2+x^2)}$$

$$\int \frac{x^2 dx}{(a^2+x^2)^2} = \frac{-x}{2(a^2+x^2)} + \frac{1}{2a} \operatorname{tg}^{-1}\left(\frac{x}{a}\right)$$

**Trigonometrijske funkcije**

$$\int \cos(x) dx = \sin(x)$$

$$\int x \cos(x) dx = \cos(x) + x \sin(x)$$

$$\int x^2 \cos(x) dx = 2x \cos(x) + (x^2 - 2) \sin(x)$$

$$\int \sin(x) dx = -\cos(x)$$

$$\int x \sin(x) dx = \sin(x) - x \cos(x)$$

$$\int x^2 \sin(x) dx = 2x \sin(x) + (2 - x^2) \cos(x)$$

**Eksponecijalne funkcije**

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$\int x e^{ax} dx = \left(\frac{x}{a} - \frac{1}{a^2}\right) e^{ax}$$

$$\int x^2 e^{ax} dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right) e^{ax}$$

$$\int x^3 e^{ax} dx = \left(\frac{x^3}{a} - \frac{3x^2}{a^2} + \frac{6x}{a^3} - \frac{6}{a^4}\right) e^{ax}$$

$$\int \sin(x) e^{ax} dx = \frac{1}{a^2+1} (a \sin(x) - \cos(x)) e^{ax}$$

$$\int \cos(x) e^{ax} dx = \frac{1}{a^2+1} (a \cos(x) + \sin(x)) e^{ax}$$

## Određeni integrali

$$\int_{-\infty}^{+\infty} e^{-a^2 x^2 + bx} dx = \frac{\sqrt{\pi}}{a} e^{\frac{b^2}{4a^2}}, \quad a > 0$$

$$\int_0^{+\infty} x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{4}$$

$$\int_0^{+\infty} \frac{\sin(x)}{x} dx = \frac{\pi}{2}$$

$$\int_0^{+\infty} \frac{\sin^2(x)}{x^2} dx = \frac{\pi}{2}$$

## Laplaceova transformacija

Jednostrana Laplaceova transformacija funkcije  $x(t)$  je:

$$\mathcal{L}(f(t)) = \int_{0^-}^{+\infty} f(t) e^{-st} dt$$

Kažemo da su  $x(t)$  i  $X(s)$  transformacijski par i pišemo  $x(t) \circ \bullet X(s)$ .

## Tablica transformacija

$$1 \circ \bullet \frac{1}{s}$$

$$t \circ \bullet \frac{1}{s^2}$$

$$e^{-at} \circ \bullet \frac{1}{s+a}$$

$$\frac{1}{b-a} (e^{-at} - e^{-bt}) \circ \bullet \frac{1}{(s+a)(s+b)}$$

$$\frac{1}{a-b} (ae^{-at} - be^{-bt}) \circ \bullet \frac{s}{(s+a)(s+b)}$$

$$\frac{1}{a} e^{-bt} \sin(at) \circ \bullet \frac{1}{(s+b)^2 + a^2}$$

$$e^{-bt} (\cos(at) - \frac{b}{a} \sin(at)) \circ \bullet \frac{s}{(s+b)^2 + a^2}$$

## Vremenski kontinuirana Fourierova transformacija

Vremenski kontinuirana Fourierova transformacija (CTFT – *Continuous-Time Fourier Transform*) funkcije  $x(t)$  je:

$$\text{CTFT}(x(t)) = X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

Inverzna transformacija je:

$$\text{ICTFT}(X(j\omega)) = x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{j\omega t} d\omega$$

Kažemo da su  $x(t)$  i  $X(j\omega)$  transformacijski par i pišemo  $x(t) \circ \bullet X(j\omega)$ .

Dovoljni (ali ne i nužni) uvjeti za postojanje transformacije funkcije  $x(t)$  su:

1. Funkcija  $x(t)$  zadovoljava Dirichletove uvjete (funkcija je ograničena s konačnim brojem maksimuma i minimuma te konačnim brojem diskontinuiteta u bilo kojem konačnom vremenskom intervalu).

2.  $\int_{-\infty}^{+\infty} |x(t)| dt < \infty$

## Svojstva Fourierove transformacije

Neka je  $x(t) \circ \bullet X(j\omega)$  i neka su  $\alpha_i$ ,  $t_0$  i  $\omega_0$  konstante. Fourierova transformacija tada zadovoljava sljedeća svojstva:

### Linearnost

$$x(t) = \sum_{i=1}^n \alpha_i x_i(t) \circ \bullet \sum_{i=1}^n \alpha_i X_i(j\omega) = X(j\omega)$$

### Dualnost

$$X(t) \circ \bullet 2\pi x(-\omega)$$

### Pomak u vremenu i frekvenciji

$$x(t - t_0) \circ \bullet X(j\omega) e^{-j\omega t_0}$$

$$x(t) e^{j\omega_0 t} \circ \bullet X(j\omega - j\omega_0)$$

### Skaliranje

$$x(\alpha t) \circ \bullet \frac{1}{|\alpha|} X\left(\frac{j\omega}{\alpha}\right)$$

### Deriviranje

$$\frac{d^n x(t)}{dt^n} \circ \bullet (j\omega)^n X(j\omega)$$

$$(-jt)^n x(t) \circ \bullet \frac{d^n X(j\omega)}{d\omega^n}$$

### Integriranje

$$\int_{-\infty}^t x(\tau) d\tau \circ \bullet \pi X(0) \delta(\omega) + \frac{X(j\omega)}{j\omega}$$

$$\pi x(0) \delta(t) - \frac{x(t)}{jt} \circ \bullet \int_{-\infty}^{\omega} X(j\xi) d\xi$$

## Konjugacija

$$x^*(t) \circ \bullet X^*(-j\omega)$$

$$x^*(-t) \circ \bullet X^*(j\omega)$$

## Konvolucija

$$\int_{-\infty}^{+\infty} x_1(\tau)x_2(t-\tau) d\tau \circ \bullet X_1(j\omega)X_2(j\omega)$$

$$x_1(t)x_2(t) \circ \bullet \frac{1}{2\pi} \int_{-\infty}^{+\infty} X_1(j\xi)X_2(j\omega-j\xi) d\xi$$

## Korelacija

$$\int_{-\infty}^{+\infty} x_1^*(\tau)x_2(t+\tau) d\tau \circ \bullet X_1^*(j\omega)X_2(j\omega)$$

$$x_1^*(t)x_2(t) \circ \bullet \frac{1}{2\pi} \int_{-\infty}^{+\infty} X_1^*(j\xi)X_2(j\omega+j\xi) d\xi$$

## Parsevalov teorem

$$\int_{-\infty}^{+\infty} x_1^*(t)x_2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X_1^*(j\omega)X_2(j\omega) d\omega$$

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega$$

## Tablica transformacija

Neka je:

$$\mu(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases}$$

$$\text{rect}(x) = \begin{cases} 1, & -\frac{1}{2} < x < \frac{1}{2} \\ 0, & \frac{1}{2} < |x| \end{cases}$$

$$\text{tri}(x) = \begin{cases} 1-|x|, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

Uz te oznake važnije transformacije su:

$$1 \circ \bullet 2\pi \delta(\omega)$$

$$\delta(t) \circ \bullet 1$$

$$\mu(t) \circ \bullet \pi \delta(\omega) + \frac{1}{j\omega}$$

$$\frac{1}{2} \delta(t) - \frac{1}{2\pi jt} \circ \bullet \mu(\omega)$$

$$\text{sgn}(t) \circ \bullet \frac{2}{j\omega}$$

$$\text{rect}\left(\frac{t}{T}\right) \circ \bullet T \text{sinc}\left(\frac{\omega T}{2\pi}\right)$$

$$\text{sinc}(at) \circ \bullet \frac{1}{a} \text{rect}\left(\frac{\omega}{2\pi a}\right)$$

$$\text{tri}\left(\frac{t}{T}\right) \circ \bullet T \text{sinc}^2\left(\frac{\omega T}{2\pi}\right)$$

$$\text{sinc}^2(at) \circ \bullet \frac{1}{a} \text{tri}\left(\frac{\omega}{2\pi a}\right)$$

$$e^{j\omega_0 t} \circ \bullet 2\pi \delta(\omega - \omega_0)$$

$$\delta(t - t_0) \circ \bullet e^{-j\omega t_0}$$

$$\sin(\omega_0 t) \circ \bullet -j\pi(\delta(\omega - \omega_0) - \delta(\omega + \omega_0))$$

$$\cos(\omega_0 t) \circ \bullet \pi(\delta(\omega - \omega_0) + \delta(\omega + \omega_0))$$

$$\sum_{i=-\infty}^{+\infty} \delta(t - iT_0) \circ \bullet \frac{2\pi}{T_0} \sum_{i=-\infty}^{+\infty} \delta\left(\frac{\omega}{2\pi} - \frac{i}{T_0}\right)$$

$$\sin(\omega_0 t) \mu(t) \circ \bullet -\frac{j\pi}{2}(\delta(\omega - \omega_0) - \delta(\omega + \omega_0)) + \frac{j\omega}{\omega_0^2 - \omega^2}$$

$$\cos(\omega_0 t) \mu(t) \circ \bullet \frac{\pi}{2}(\delta(\omega - \omega_0) + \delta(\omega + \omega_0)) + \frac{j\omega}{\omega_0^2 - \omega^2}$$

$$e^{-at} \mu(t) \circ \bullet \frac{1}{a + j\omega}, \quad a > 0$$

$$te^{-at} \mu(t) \circ \bullet \frac{1}{(a + j\omega)^2}, \quad a > 0$$

$$t^2 e^{-at} \mu(t) \circ \bullet \frac{2}{(a + j\omega)^3}, \quad a > 0$$

$$t^3 e^{-at} \mu(t) \circ \bullet \frac{6}{(a + j\omega)^4}, \quad a > 0$$

$$e^{-a|t|} \circ \bullet \frac{2a}{a^2 + \omega^2}$$

$$e^{-\frac{t^2}{2a^2}} \circ \bullet a\sqrt{2\pi} e^{-a^2\omega^2/2}$$

## Vremenski kontinuiran Fourierov red

Vremenski kontinuiran Fourierov red (CTFS – *Continuous-Time Fourier Series*) periodične funkcije  $x(t)$  s periodom  $T_0$  je:

$$\text{CTFS}_{T_0}(x(t)) = X_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-j\omega_0 kt} dt$$

Inverzna transformacija je:

$$\text{ICTFS}_{T_0}(X_k) = x(t) = \sum_{k=-\infty}^{+\infty} X_k e^{j\omega_0 kt}$$

Pri tome je  $\omega_0 = \frac{2\pi}{T_0}$ . Kažemo da su  $x(t)$  i  $X_k$  transformacijski par i pišemo  $x(t) \circ \bullet X_k$ .

## Vremenski diskretna Fourierova transformacija

Vremenski diskretna Fourierova transformacija (DTFT – *Discrete-Time Fourier Transform*) niza  $x(n)$  je:

$$\text{DTFT}(x(n)) = X(e^{j\Omega}) = \sum_{n=-\infty}^{+\infty} x(n)e^{-j\Omega n}$$

Inverzna transformacija je:

$$\text{IDTFT}(X(e^{j\Omega})) = x(n) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} X(e^{j\Omega})e^{j\Omega n} d\Omega$$

Niz  $x(n)$  i njegov spektar  $X(e^{j\Omega})$  čine transformacijski par  $x(n) \circ \bullet X(e^{j\Omega})$ .

Dovoljan (ali ne i nužni) uvjet za postojanje transformacije niza  $x(n)$  je apsolutna sumabilnost:

$$\sum_{n=-\infty}^{+\infty} |x(n)| < \infty$$

## Svojstva vremenski diskretne Fourierove transformacije

Neka je  $x(n) \circ \bullet X(e^{j\Omega})$  i neka su  $\alpha_i$ ,  $n_0$  i  $\Omega_0$  konstante. Vremenski diskretna Fourierova transformacija tada zadovoljava sljedeća svojstva:

### Linearnost

$$x(n) = \sum_{i=1}^n \alpha_i x_i(n) \circ \bullet \sum_{i=1}^n \alpha_i X_i(e^{j\Omega}) = X(e^{j\Omega})$$

### Pomak u vremenu i frekvenciji

$$x(n - n_0) \circ \bullet X(e^{j\Omega})e^{-j\Omega n_0}$$

$$x(n)e^{j\Omega_0 n} \circ \bullet X(e^{j\Omega - j\Omega_0})$$

### Deriviranje i diferenciranje

$$\Delta x(n) \circ \bullet (e^{j\Omega} - 1)X(e^{j\Omega})$$

$$n^i x(n) \circ \bullet j^i \frac{d^i X(e^{j\Omega})}{d\Omega^i}$$

### Sumiranje

$$\sum_{i=-\infty}^n x(i) \circ \bullet \frac{1}{1 - e^{-j\Omega}} X(e^{j\Omega})$$

## Konjugacija

$$x^*(n) \circ \bullet X^*(e^{-j\Omega})$$

$$x^*(-n) \circ \bullet X^*(e^{j\Omega})$$

## Konvolucija

$$\sum_{i=-\infty}^{+\infty} x_1(i)x_2(n-i) \circ \bullet X_1(e^{j\Omega})X_2(e^{j\Omega})$$

$$x_1(n)x_2(n) \circ \bullet \frac{1}{2\pi} \int_{-\pi}^{+\pi} X_1(e^{j\xi})X_2(e^{j\Omega-j\xi}) d\xi$$

## Parsevalov teorem

$$\sum_{n=-\infty}^{+\infty} x_1^*(n)x_2(n) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} X_1^*(e^{j\Omega})X_2(e^{j\Omega}) d\Omega$$

$$\sum_{n=-\infty}^{+\infty} |x(n)|^2 = \frac{1}{2\pi} \int_{-\pi}^{+\pi} |X(e^{j\Omega})|^2 d\Omega$$

## Relacije simetričnosti

Neka je  $x(n)$  čisto realan niz i neka je  $x(n) \circ \bullet X(e^{j\Omega})$ . Tada je:

$$\frac{1}{2}(x(n) + x(-n)) \circ \bullet \text{Re}(X(e^{j\Omega}))$$

$$\frac{1}{2}(x(n) - x(-n)) \circ \bullet j \text{Im}(X(e^{j\Omega}))$$

Također vrijedi:

$$X(e^{j\Omega}) = X^*(e^{-j\Omega})$$

$$\text{Re}(X(e^{j\Omega})) = \text{Re}(X(e^{-j\Omega}))$$

$$\text{Im}(X(e^{-j\Omega})) = -\text{Im}(X(e^{j\Omega}))$$

## Tablica transformacija

$$\delta(n) \text{ --- } \bullet 1$$

$$1 \text{ --- } \bullet \sum_{i=-\infty}^{+\infty} 2\pi\delta(\Omega + 2\pi i)$$

$$e^{j\Omega_0 n} \text{ --- } \bullet \sum_{i=-\infty}^{+\infty} 2\pi\delta(\Omega - \Omega_0 + 2\pi i)$$

$$\mu(n) \text{ --- } \bullet \frac{1}{1 - e^{-j\Omega}} + \sum_{i=-\infty}^{+\infty} \pi\delta(\Omega + 2\pi i)$$

$$a^n \mu(n) \text{ --- } \bullet \frac{1}{1 - ae^{-j\Omega}}, \quad |a| < 1$$

$$na^n \mu(n) \text{ --- } \bullet \frac{ae^{j\Omega}}{(e^{-j\Omega} - a)^2}, \quad |a| < 1$$

$$\sin(\Omega_0 n) \text{ --- } \bullet \sum_{i=-\infty}^{+\infty} j\pi(\delta(\Omega + \Omega_0 + 2\pi i) - \delta(\Omega - \Omega_0 + 2\pi i))$$

$$\cos(\Omega_0 n) \text{ --- } \bullet \sum_{i=-\infty}^{+\infty} \pi(\delta(\Omega + \Omega_0 + 2\pi i) + \delta(\Omega - \Omega_0 + 2\pi i))$$

$$a^n \sin(\Omega_0 n) \mu(n) \text{ --- } \bullet \frac{ae^{j\Omega} \sin(\Omega_0)}{e^{2j\Omega} - 2ae^{j\Omega} \cos(\Omega_0) + a^2}, \quad |a| < 1$$

$$a^n \cos(\Omega_0 n) \mu(n) \text{ --- } \bullet \frac{e^{j\Omega}(e^{j\Omega} - a \cos(\Omega_0))}{e^{2j\Omega} - 2ae^{j\Omega} \cos(\Omega_0) + a^2}, \quad |a| < 1$$

## Vremenski diskretan Fourierov red

Vremenski diskretan Fourierov red (DTFS – *Discrete-Time Fourier Series*) periodičnog niza  $x(n)$  perioda  $N$  je:

$$\text{DTFS}_N(x(n)) = X_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n)e^{-2\pi jkn/N}$$

Inverzna transformacija je:

$$\text{IDTFS}_N(X_k) = x(n) = \sum_{k=0}^{N-1} X_k e^{2\pi jkn/N}$$

Niz  $x(n)$  i njegov spektar  $X_k$  čine transformacijski par  $x(n) \text{ --- } \bullet X_k$ .

## Diskretna Fourierova transformacija

Diskretna Fourierova transformacija (DFT – *Discrete Fourier Transform*) konačnog niza  $x(n)$  duljine  $N$  je:

$$\text{DFT}_N(x(n)) = X(k) = \sum_{n=0}^{N-1} x(n)W_N^{nk}, \quad 0 \leq k < N$$

Pri tome je  $W_N^{nk} = e^{-2\pi jnk/N}$ . Inverzna transformacija je:

$$\text{IDFT}_N(X(k)) = x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)W_N^{-nk}, \quad 0 \leq n < N$$

Niz  $x(n)$  i njegov spektar  $X(k)$  čine transformacijski par  $x(n) \text{ --- } \bullet X(k)$ .

## Svojstva diskretne Fourierove transformacije

Neka je  $x(n) \text{ --- } \bullet X(k)$  i neka su  $\alpha_i$ ,  $n_0$  i  $k_0$  konstante. DFT tada zadovoljava sljedeća svojstva:

### Linearnost

$$x(n) = \sum_{i=1}^n \alpha_i x_i(n) \text{ --- } \bullet \sum_{i=1}^n \alpha_i X_i(k) = X(k)$$

### Dualnost

$$X(n) \text{ --- } \bullet Nx(\langle -k \rangle_N)$$

### Cirkularni pomak u vremenu i frekvenciji

$$x(\langle n - n_0 \rangle_N) \text{ --- } \bullet X(k)W_N^{kn_0}$$

$$x(n)W_N^{k_0 n} \text{ --- } \bullet X(\langle k - k_0 \rangle_N)$$

### Cirkularna konvolucija

$$\sum_{i=0}^{N-1} x_1(i)x_2(\langle n - i \rangle_N) \text{ --- } \bullet X_1(k)X_2(k)$$

$$x_1(n)x_2(n) \text{ --- } \bullet \frac{1}{N} \sum_{i=0}^{N-1} X_1(i)X_2(\langle k - i \rangle_N)$$

### Parsevalova relacija

$$\sum_{n=0}^{N-1} x_1^*(n)x_2(n) \text{ --- } \bullet \frac{1}{N} \sum_{k=0}^{N-1} X_1^*(k)X_2(k)$$

$$\sum_{n=0}^{N-1} |x(n)|^2 \text{ --- } \bullet \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2$$

## Z-transformacija

Z-transformacija niza  $f(n)$  je:  $\mathcal{Z}(f(n)) = \sum_{n=0}^{+\infty} f(n)z^{-n}$

## Svojstva Z transformacije

Neka je  $\mathcal{Z}(f(n)) = F(z)$  i  $\mathcal{Z}(g(n)) = G(z)$ . Tada vrijedi:

### Linearnost

$$f(n) = \sum_{i=1}^n \alpha_i f_i(n) \circ \bullet \sum_{i=1}^n \alpha_i F_i(z) = F(z)$$

### Pomak

$$f(n+1) \circ \bullet zF(z) - zf(0)$$

$$f(n+m) \circ \bullet z^m F(z) - \sum_{i=0}^{m-1} f(i)z^{m-i}$$

$$f(n-1) \circ \bullet \frac{1}{z}F(z) + f(-1)$$

$$f(n-m) \circ \bullet z^{-m}F(z) + \sum_{i=0}^{m-1} f(i-m)z^{-i}$$

### Skaliranje

$$a^n f(n) \circ \bullet F\left(\frac{z}{a}\right)$$

### Diferenciranje i deriviranje

$$\Delta f(n) \circ \bullet (z-1)F(z)$$

$$nf(n) \circ \bullet -z \frac{dF(z)}{dz}$$

### Konvolucija

$$\sum_{i=0}^{+\infty} f(i)g(n-i) \circ \bullet F(z)G(z)$$

## Tablica transformacija

$$\delta(n) \circ \bullet 1$$

$$\delta(n-m) \circ \bullet z^{-m}$$

$$n \circ \bullet \frac{z}{(z-1)^2}$$

$$1^n \circ \bullet \frac{1}{1-z^{-1}} = \frac{z}{z-1}$$

$$a^n \circ \bullet \frac{1}{1-az^{-1}} = \frac{z}{z-a}$$

$$(n+1)a^n \circ \bullet \frac{z^2}{(z-a)^2}$$

$$\frac{(n+1)(n+2)}{2!} a^n \circ \bullet \frac{z^3}{(z-a)^3}$$

$$\frac{(n+1)(n+2)\dots(n+m-1)}{(m-1)!} a^n \circ \bullet \frac{z^m}{(z-a)^m}$$

$$\frac{n(n-1)(n-2)\dots(n-m+1)}{m!} a^{n-m} \circ \bullet \frac{z}{(z-a)^{m+1}}$$

$$a^n - \delta(n) \circ \bullet \frac{a}{z-a}$$

$$\sin(an) \circ \bullet \frac{z \sin(a)}{z^2 - 2z \cos(a) + 1}$$

$$\cos(an) \circ \bullet \frac{z^2 - z \cos(a)}{z^2 - 2z \cos(a) + 1}$$

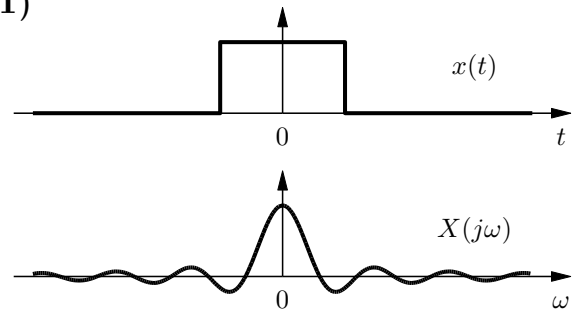
## Pregled Fourierovih transformacija

### Vremenski kontinuirana Fourierova transformacija (CTFT)

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega)e^{j\omega t} d\omega$$

$$E = \int_{-\infty}^{+\infty} x(t)x^*(t) dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega)X^*(j\omega) d\omega$$

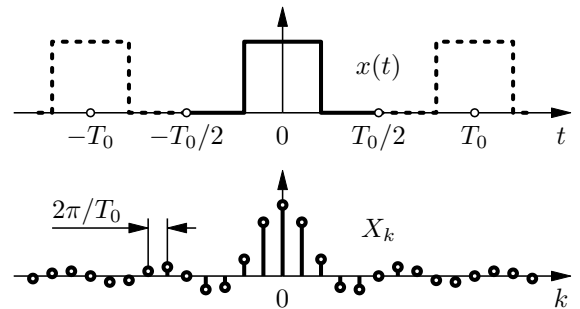


### Vremenski kontinuiran Fourierov red (CTFS)

$$X_k = \frac{1}{T_0} \int_{T_0} x(t)e^{-j\omega_0 kt} dt$$

$$x(t) = \sum_{k=-\infty}^{+\infty} X_k e^{j\omega_0 kt}$$

$$P = \frac{1}{T_0} \int_{T_0} x(t)x^*(t) dt = \sum_{k=-\infty}^{+\infty} X_k X_k^*$$

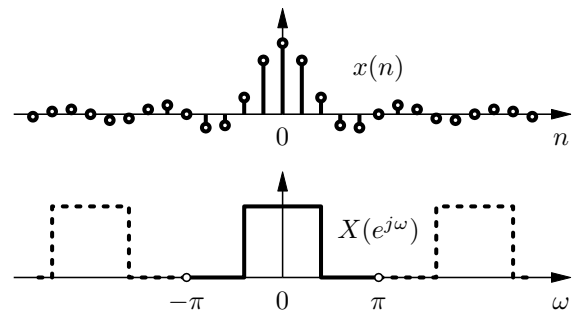


### Vremenski diskretna Fourierova transformacija (DTFT)

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x(n)e^{-j\omega n}$$

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n} d\omega$$

$$E = \sum_{n=-\infty}^{+\infty} x(n)x^*(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})X^*(e^{j\omega}) d\omega$$

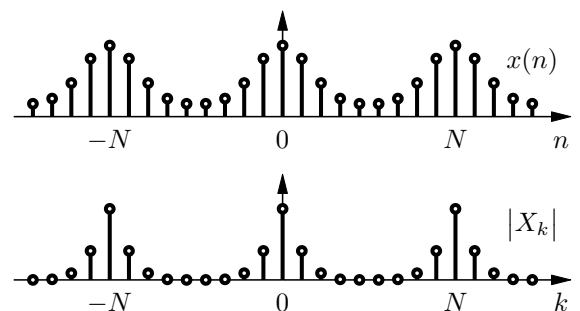


### Vremenski diskretan Fourierov red (DTFS)

$$X_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n)e^{-2\pi jkn/N}$$

$$x(n) = \sum_{k=0}^{N-1} X_k e^{2\pi jkn/N}$$

$$P = \frac{1}{N} \sum_{n=0}^{N-1} x(n)x^*(n) = \sum_{k=0}^{N-1} X_k X_k^*$$

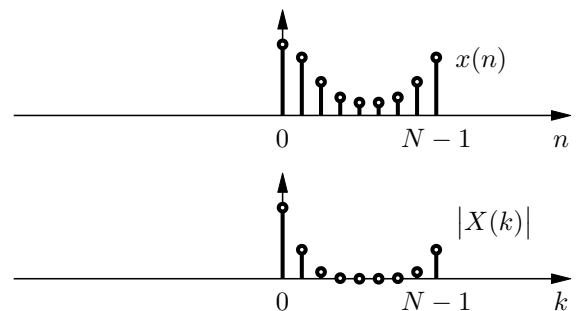


### Diskretna Fourierova transformacija (DFT)

$$X(k) = \sum_{n=0}^{N-1} x(n)W_N^{kn}, \quad 0 \leq k \leq N-1$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)W_N^{-kn}, \quad 0 \leq n \leq N-1$$

$$\sum_{n=0}^{N-1} x_1(n)x_2^*(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_1(k)X_2^*(k)$$



## Određivanja početnih uvjeta

Za sustav opisan diferencijalnom jednačbom

$$y^{(N)}(t) + a_1 y^{(N-1)}(t) + \dots + a_{N-1} y^{(1)}(t) + a_N y(t) = b_0 u^{(N)}(t) + b_1 u^{(N-1)}(t) + \dots + b_{N-1} u^{(1)}(t) + b_N u(t)$$

potrebno je odrediti početne uvjete  $y(0^+)$ ,  $y'(0^+)$ ,  $y''(0^+)$ ,  $\dots$ ,  $y^{(N-1)}(0^+)$  iz onih u  $0^-$ . Ako pobuda ne sadrži Diracovu distribuciju rješavamo sustav jednačbi:

$$\begin{aligned}\Delta y &= b_0 u(0^+) \\ \Delta y^{(1)} + a_1 \Delta y &= b_0 u^{(1)}(0^+) + b_1 u(0^+) \\ \Delta y^{(2)} + a_1 \Delta y^{(1)} + a_2 \Delta y &= b_0 u^{(2)}(0^+) + b_1 u^{(1)}(0^+) + b_2 u(0^+)\end{aligned}$$

$$\Delta y^{(N-1)} + a_1 \Delta y^{(N-2)} + \dots + a_{N-1} \Delta y = b_0 u^{(N-1)}(0^+) + \dots + b_{N-2} u^{(1)}(0^+) + b_{N-1} u(0^+)$$

Pri tome je  $\Delta y^{(i)} = y^{(i)}(0^+) - y^{(i)}(0^-)$ .

Ako je pobuda  $u(t) = \delta(t)$  onda je  $y^{(N-1)}(0^+) = y^{(N-1)}(0^-) + 1$ , a ostali početni uvjeti se ne razlikuju.