**White Paper**

# Optimal Power Flow— Basic Requirements for Real-Life Problems and their Solutions

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## FOREWORD

Optimal power flow (OPF) calculations are increasingly critical to modern power systems operation-control, markets and planning. However, it is still not easy to obtain reliable OPF solutions that are fully usable in practical power systems engineering. Better methods and software are continually being pursued.

This paper discusses some of the calculation and solution requirements from the viewpoints of:

(a) Users seeking better OPF tools, and

(b) Researchers and developers trying to provide such tools.

The paper is based on four decades of experience in developing and supplying the relevant OPF methods and software to industry all over the world. It makes no attempt at being a comprehensive subject review or at providing a compilation of references.



## 1. Introduction

### 1.1. General

Optimal power flow is one of the fundamental classes of static power system network calculations[1]. Over the last 50 years it has been the subject of thousands of small-scale OPF research projects, mostly based on problem formulations that are simplified to the point of being mathematical, rather than power systems engineering analysis, exercises. There have also been perhaps dozens of large-scale attempts to develop reliable, versatile, efficient, reasonably accurate, industrial OPF calculation software. However, these have not proved to be easy or completely successful.

Today, OPF is increasingly central to economy-security processes in power system operations and control, markets and planning. The OPF calculation is an application on its own or it can be a sub-

[1] Dommel & Tinney, "Optimal power flow solutions", IEEE Trans. Power App. Syst., Oct. 1968, pp.1866-1876.

function of a larger problem such as security constrained unit commitment. There are great incentives to develop industrial-quality OPF programs that provide the user with better solutions. At the time of writing, various such multi-year initiatives are in progress around the globe.

The biggest challenge in OPF development is to anticipate the modeling and solution requirements of the prospective power systems users. Many of these requirements contribute greatly to the complexity of the problem.

OPF basics are covered in most power systems analysis text books. References [1]–[6] are samples of OPF subject reviews, with extensive bibliographies. Reference [6], covering many modern OPF issues that overlap with those discussed here, is particularly noteworthy.

### 1.2. Scope of this Paper

OPF calculations, whether for application in online control, in operations, in operational planning, in markets or in planning, involve many solution function, quality and performance issues. In this paper, we examine requirements for traditional deterministic OPF problems with a single static objective. The controllable variables are optimized subject to static constraints imposed mostly by the operation of the transmission grid, modeled in standard accurate AC manner.

The paper mostly addresses general OPF analysis related questions. It discusses basic requirements in practical OPF problem formulations and solutions. It is not concerned with specific OPF applications or algorithms.

*The vast majority of practical OPF problems involve security (contingency) constraints.* Therefore, in our usage, we do not distinguish separately between the terms OPF and SCOPF (Security Constrained OPF). Note that security constraints, which have a profound influence on the choice and performance of solution methodologies, have been ignored by most small-scale OPF R&D over the years.

It is clearly impossible to address all relevant aspects of all OPF problems. Here we will try to cover a series of <u>general OPF issues</u> that in our experience are important for prospective or current OPF users and developers.

## 2. OPF BASIC STATEMENT

### 2.1. Basic Problem Statement

A version of the well-known OPF problem statement, expressed in terms of the power system control (or decision) variables $\boldsymbol{U}$ and state variables $\boldsymbol{X}$, is as follows:

Minimize a scalar function:

$$\boldsymbol{f}(\boldsymbol{U}^{o}, \boldsymbol{X}^{o}) \tag{1}$$

subject to equality and inequality constraints:

$$\boldsymbol{G}^{i}(\boldsymbol{U}^{i}, \boldsymbol{X}^{i}) = \boldsymbol{0} \tag{2}$$

$$\boldsymbol{H}^{i}(\boldsymbol{U}^{i}, \boldsymbol{X}^{i}) \leq \boldsymbol{0} \tag{3}$$

$$\boldsymbol{U}^{i} = \boldsymbol{M}^{i}(\boldsymbol{U}^{o}) \tag{4}$$

for $i = 0 \ldots n$, where $i = 0$ denotes the pre-contingency (base-case) state and otherwise denotes a post-contingency state. Functions $\boldsymbol{G}$ mostly comprise the sparse power flow equations. Functions $\boldsymbol{H}$ are mostly very sparse constraints on flows, voltages, etc.

### 2.2. Comments on Problem Statement

In the above problem statement (1) to (4), it has always been tempting to assume that functions $\boldsymbol{f}$, $\boldsymbol{G}$ and $\boldsymbol{H}$ are smooth and analytic, and that $\boldsymbol{M}$ is linear. In this case, the OPF is a nonlinear programming

(NLP) problem, for which many solution methods are available, including those found in powerful modern general-purpose mathematical optimization packages.

However, typical real-life OPF problems are more complicated. Special power-system-specific solution techniques are needed to handle the very large numbers of constraints and a range of quite onerous solution-seeking and modeling peculiarities. In particular:

- Limits on power system apparatus and system operation can cause the models and their sensitivities to undergo sharp state-dependent discontinuities, as will be discussed a number of times later.
- A practical OPF solution process searches for a useful result under the guidance of objective, control and constraint priorities and rules. Of particular importance is the triggering of such rules when the problem constraints are infeasible, near-infeasible, or too expensive to enforce. The intelligent implementation of such rules may involve backtracking during the solution process (completely restarting the problem each time, and losing valuable information thus-far obtained, is not normally appropriate). Sometimes "pseudo-commitment" features are formulated, where an item of apparatus (including a generator) becomes connected or disconnected as a function of a threshold marginal cost.
- Solution convergence is an acute problem when, at any given point in the solution, the constraints represent a physically unstable or ill-conditioned operating state of the power system, particularly in post-contingency mode. This is a severely under-addressed issue, and again suggests the use of back-tracking techniques during the solution process.

### 2.3. Comments on the OPF Solution Process

In this paper we do not deal with specific solution methods, but Figure 1 illustrates by far the most common type of iterative OPF calculation procedure.

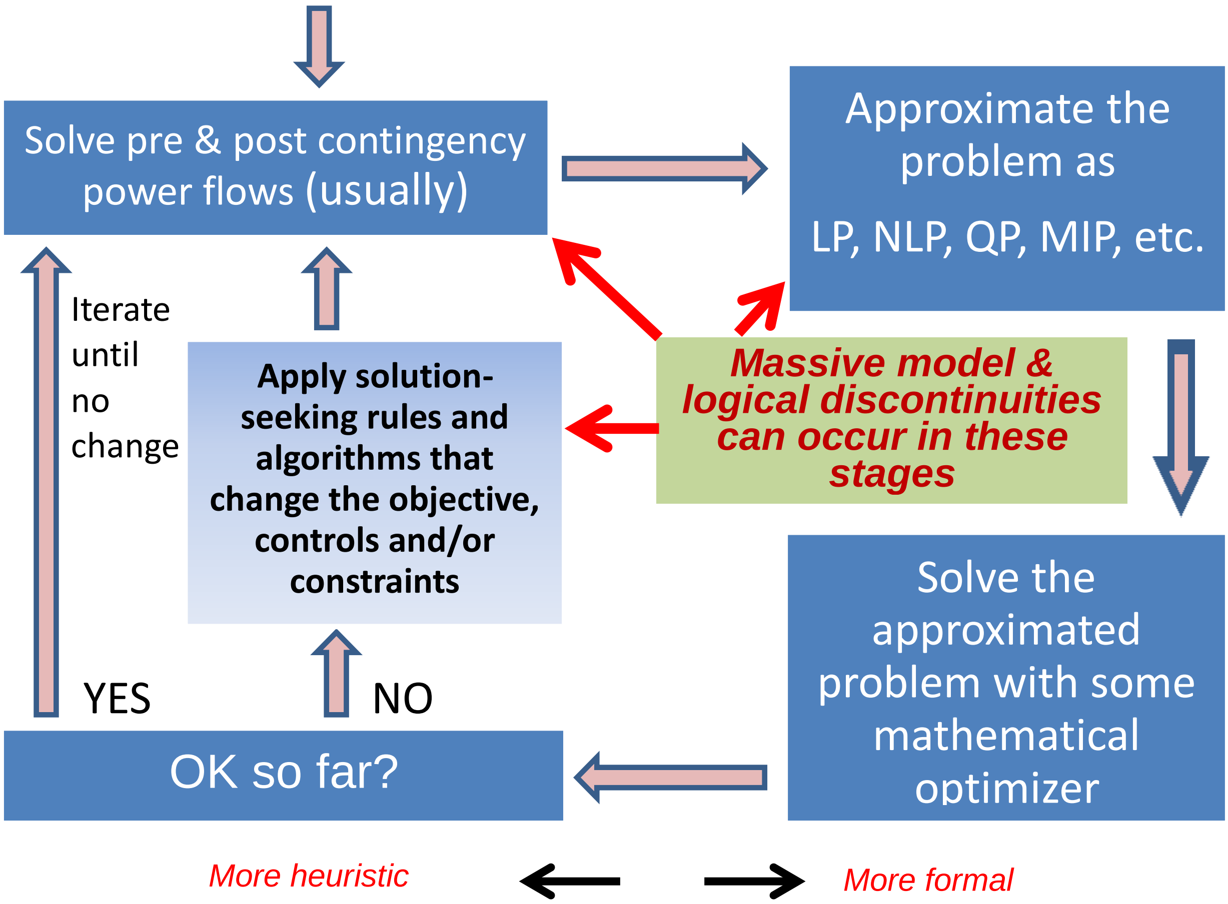


***Figure 1: A typical iterative OPF solution process***

Some general observations about this calculation process and the nature of the solution can be made as follows:

- The realistic OPF modeling of power system apparatus, grid operation and other constraints, plus objectives and priorities, is complicated, intricate and non-smooth.
- Every efficient optimizer is designed to handle problems with specific mathematical structures (e.g. linear, quadratic, non-linear or mixed-integer programming); therefore the OPF problem has to be iteratively re-approximated in the relevant form outside the optimizer.
- At each OPF iteration, the sensitivities between the objective, control and constraint functions can and usually do change abruptly. In a non-trivial power system, some of these changes are difficult or impossible to formulate analytically, even in any sophisticated mixed-integer manner.
- In any typical OPF calculation, the central constrained optimizer will be entered cyclically. This optimizer must very flexibly adapt itself to such frequent discontinuous changes in a hot-start manner.
- Because a realistic OPF problem is non-smooth, convergence is rarely asymptotic, and the eventual solution is typically path dependent. At best, the solution to an OPF problem can be taken as exact only for its very last approximation.
- The number of OPF constraints can be vast (up to hundreds of millions). A critical aspect of the entire OPF solution is one or more "outer loop" processes that are designed to identify those few constraints and contingency cases that might bind in the eventual solution. This is an extremely important heuristic power-system-specific part of the OPF solution process. It contains a variety of major pitfalls.
- An optimizer that handles nonlinear constraints may be at an advantage as far as the pre-contingency network model is concerned. But much of this advantage is lost as soon as post-contingency constraints are introduced, because for non-small systems the only practical way to impose these latter is to successively linearize them (usually non-smoothly, as above).
- It is unlikely that useful theories of convergence or global optimality can be developed for non-trivial real-life OPF problem formulations.

# 3. SECURITY CONCEPTS IN OPF

## 3.1. Security Levels

"N-1" security is frequently quoted in power systems operations and planning. It is defined as a power system's ability to survive intact following a single contingency. "N-2" security refers to two simultaneous contingencies, and so on. So-called "N-1-1" security refers to one contingency, and then, after the initial system response, a second contingency, whether a consequence of the first or not.

A great deal hinges on the notion of "contingency". OPF R&D literature too often treats a contingency as a single (easy to simulate) branch outage. In practice, power system engineers study the vulnerabilities of their systems and they develop probability-based contingency lists. A single contingency is a plausible "system event" that can involve anything from one to many simultaneous changes to the pre-contingency power system. These changes can include:

- The combined outages or re-energization of series and shunt branches, generators, loads, and other facilities.
- Complete bus outages (e.g. due to stuck breakers).
- Bus splitting or merging that changes the electrical topology.
- Electrical islanding, where each survivable island has to be modeled and securely optimized.
- Pre-programmed generator rescheduling, within generator limits.
- Conditional automatic and possibly cascading relay actions that are triggered by the initial post-contingency branch loadings, voltages, interface flows, etc.
- Post-contingency remedial (corrective) control actions actuated by the operator or by the central EMS software.

The pre- and post-contingency models of the power system can have profoundly non-analytic relationships with each other. The most obvious example is when modeling automatic generator responses (within unit MW limits) following a contingency that substantially alters the system MW balance—generator or load outages, network islanding, etc. Then function $\boldsymbol{M}^{\boldsymbol{i}}$ in (4) is an algorithm with no differentiable form[2]. Similar considerations apply in the voltage-VAR sub-problem.

## 3.2. Preventive Security

In a preventively secure pre-contingency (base case) operating state, the power system will suffer no constraint violations following any single contingency and the resulting response of automatic controls. This security criterion is in need of revision because not all constraints (a) are of equal likelihood, and (b) have equal operational or economic impacts. However, it is the current norm, and can largely be handled by current OPF technology.

## 3.3. Corrective Security

### *3.3.1. Concept*

This is a current smart-grid "hot topic". It is not implemented anywhere yet, but eventually it promises considerable economy in system operation (and planning, in the long run?). Among other things, it can reduce the amount and cost of short-term reserve, particularly in the face of volatile renewable generation, and as large-scale locational control of demand becomes more viable.

When constraint violations occur following any contingency and the consequent response of automatic controls, the pre-contingency state is deemed "correctively secure" if all such violations can be removed within a specified amount of time by optimal (in a defined sense) centralized remedial action. The concept applies primarily to overload-tolerant thermal constraints, noting that it is not uncommon for transmission line short-term ratings to be 30-40% higher than their long-term values. Such ratings are increasingly determined dynamically.

Corrective security represents a trade-off between operating savings and security margins. It becomes most effective and palatable in short-term operation and control. The numbers and types of post-contingency corrective actions depend on how each remedial action can be applied—whether manually or automatically. Typically, a small prioritized number of corrective actions will be sought.

In this approach, a security constrained OPF calculation will handle a mix of preventive and corrective constraints. Some constraints, particularly on important voltages, will remain enforced preventively. Note that a quantity may have its longer-term limit imposed correctively and at the same time have its shorter-term limit imposed preventively.

As a rather radical departure from preventive security, implementing the approach will no doubt be controversial. It is capable of being introduced practically in a very incremental, try-it-and-see manner, e.g. initially allowing only small post-contingency corrections, with restricted limits.

### *3.3.2. Calculations*

In the corrective security mode, the OPF has to optimize the base-case (pre-contingency) operating state subject to the results of optimized post-contingency corrective action. These calculations are sophisticated and are subject to further R&D. Although this paper is mostly about solution requirements rather than specific algorithms, corrective security is of sufficient interest to warrant some comments on potential solution methods.

Many years ago, it was shown that corrective security dispatch can be performed using Benders decomposition[3]. The strongest point of this approach is that, subject to good convergence, it remains efficient for large power system networks. Very large-scale practical testing is needed.

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[2] Simplified OPF formulations frequently seek analytical convenience by ignoring such modeling details and only considering non-islanding branch outage contingencies—generators respond without MW limits and/or the island reference bus absorbs the post-contingency MW imbalance. This is not sufficiently general or accurate for practical purposes.

[3] Monticelli, Pereira & Granville, "Security-constrained optimal power flow with post-contingency corrective rescheduling", IEEE Trans. Power Syst., Feb. 1987, pp. 175-180.

A much more direct approach is also possible. The pre-contingency case and all those post-contingency cases with potentially binding constraints are iteratively approximated sparsely and submitted to the central optimizer as one huge simultaneous problem, where each contingency state has coupling constraints to the pre-contingency state. As usual, throughout the solution process the very large sparse lumped set of approximated equations and constraints is subject to discontinuous changes, and the contingency cases in the set will tend to vary.

With today's computing powers, this approach seems limited to: (a) small power systems, or (b) systems whose models can be reduced on-the-fly without sacrifice of accuracy and reliability[4], and (c) problems where few contingencies have potentially binding constraints [note that more contingency cases will contribute binding constraints compared with preventive security].

### 3.4. Preventive-Corrective Hybrid Security

This hybrid treats some contingencies preventively and others correctively. Its potential for operational economy is somewhere in between these two security levels. Its solution process is very much simpler than for true corrective security as above, and can be outlined briefly as follows.

After each optimization pass of the OPF calculation, post-contingency corrective control (remedial action) is simulated on each contingency case with overload-tolerant violations. If the violations can be relieved within the prescribed time, the case is skipped. Otherwise, the case's relevant constraints become enforced in the traditional preventive manner. This is likely to result in fewer binding contingencies than in preventive security.

However, this "preventive-corrective" hybrid approach has a major drawback—different contingency cases end up with totally different levels of security, depending somewhat randomly on the specific network operating condition.

To illustrate, consider at a certain stage of the OPF calculation:

(a) A contingency case with one or more constraint violations that are marginally too large to be alleviated by post-contingency corrective action. In the hybrid approach, the violated post-contingency constraints will be enforced preventively—this normally involves significant (costly) pre-contingency re-dispatch. [By contrast, in the corrective security approach of Section 3.3 above, pre-contingency operation would tend to be adjusted only slightly, to bring the post-contingency state to a correctable condition.]

(b) A case that is identical to (a) except that the violation values are very slightly lower, such that the case is pronounced post-contingency correctable. This contingency case and constraints are therefore skipped.

Operationally, these cases are virtually the same, but the hybrid approach handles situation (a) at the high-security preventive level, leaving situation (b) much more vulnerable with only borderline corrective security. It is easy to see that a small variation in system dispatch could change, and even reverse, the security treatments of the two cases. Such inconsistencies can be mitigated by using different corrective and preventive constraint limits, but this seems to be a difficult tuning problem.

The underlying issue is that, unlike preventive and corrective security in the previous sections, the preventive-corrective hybrid does not represent a formal system security criterion. Rather, it can be viewed as an ad-hoc opportunistic strategy for short-term operation, to extract extra economy from a power system that remains planned for preventive security. Therefore it could be difficult or impossible to incorporate the hybrid security concept into future system operational planning or even planning procedures, or perhaps even into day-ahead security-constrained unit commitment.

Depending on whether backoff in (a) above is allowed during the OPF calculation, a challenge is to avoid oscillation between preventively and correctively secure contingency cases.

[4] Among other OPF challenges, this is outlined in the review paper Ref. [6].

# 4. FORMULATIONS

## 4.1. Objective Functions

A typical OPF objective function is "separable"—for example the sum of individual generator and demand “cost” curves. Traditional text-book economic dispatch normally assumes convex and piecewise polynomial curves. Non-convex objectives are increasingly encountered. In any case, objective function convexity by no means assures problem convexity.

Today, economic dispatch is only one of many common scalar OPF objectives, which include least-squares or least-number of control shifts, least-cost VAR installation, and financial transmission rights auction valuation. In modern markets, and in least-number-of-shift problems, the cost curves are very often linear or piecewise linear.

The objective function strongly affects the overall problem characteristics and the choice of solution approach. Some algorithms behave best with strong objective function convexity. Some are poorly suited to objective function discontinuities (e.g. piecewise, or piecewise in the first derivative). Loss objectives, which are highly nonlinear and non-separable, respond better with second-order algorithms. Other algorithms work best with linear objectives. Conversely, cost curves with only slight nonlinearity are troublesome for many optimization algorithms.

Devising a general-purpose OPF solution approach that handles a wide range of objectives and other problem aspects is difficult. Moreover, certain objectives can become quite complicated, as for instance with combined-cycle plants and co-optimization.

## 4.2. Multi-Component Objectives

True multi-objective OPF problems have been formulated, but industry applications have been few. Regardless, in order to arrive at practically useful solutions, the main objective in every OPF calculation typically has to be augmented (before and/or during the calculation) with additional functions for purposes such as:

- Including the optimization of controls for which clear costs cannot be ascribed.
- Discouraging controls or control targets from moving from their initial or scheduled values.
- Penalizing soft constraint violations.
- Preventing limit violations via barrier functions (e.g. as in interior point methods).
- Overcoming degeneracy.
- Invoking priorities.
- Encouraging appropriate sharing between different controls.
- Suppressing oscillations.
- Promoting control discreteness.

Expressing different objective function components such as these on a common "cost" scale is extremely difficult and almost always somewhat arbitrary. For instance, when minimizing generator MW costs, how to monetize the cost of moving a phase shifter angle? Or when minimizing the rescheduling of VQ controls, how to scale minimum-shift cost curves that give appropriate relative weightings to fundamentally different quantities such as generator voltages, shunt admittances, and transformer taps?

## 4.3. Matching Objectives, Controls and Constraints

### *4.3.1. General*

In formulating any OPF problem, it is necessary to consider carefully which optimized controls and constraints are appropriate for the objective(s) and applications addressed.

When a MW-only dc-type network model is used, this is relatively straightforward—VQ controls are then absent and VQ constraints, if any, must be imposed in the form of surrogate MW limits.

More generally, the network is described by an AC model. Then a variety of objective-control-constraint couplings become possible. Engineering considerations are critical to the mix of controls and constraints that are practical for each application and objective.

Obviously, for example, in optimizing a MW-related objective (e.g. generation cost) using P controls, the main constraints will be those "P constraints" that are expected to have significant coupling with the controls. There is often scope for including VQ constraints in such a problem, but this needs considerable care—it is not practical to enforce VQ limits by heroic, highly uneconomic P control rescheduling, when such limits are more realistically handled locally by VQ controls. Operationally, when starting from a given state, this situation is often addressed simply by requiring that P control optimization does not make the constrained VQ quantities worse.

Conversely, by the very nature of AC transmission, optimized VQ controls normally have little physical ability to enforce non-local P constraints such as MVA and ampere flow limits—if this "fact of life" is ignored, the optimization can force many of these VQ controls to their limits, and produce a power system operating state with unacceptable voltage profiles and little voltage-VAR control flexibility and reserve.

The next subsections continue to discuss these types of critically important considerations.

#### *4.3.2. Dispatching Ineffective Controls*

When one or more violating constraints have no controls electrically sensitive to them, the OPF problem is infeasible. When such constraints only have marginally sensitive controls, it usually makes no engineering sense to try to enforce them using these controls—the result would be to force the controls to undergo huge, uneconomical, and perhaps ultimately unsuccessful shifts from the values that they would otherwise have. Since it is not always possible to take care of such problems in the OPF formulation itself, these situations need to be detected and identified during the OPF solution process.

Decisions about how to proceed then have to be taken—for instance (a) ignore constraints, (b) expand their limits, and/or (c) invoke priority rules that might include committing more control resources. Designating a control as ineffective is normally not just a one-to-one relationship between itself and a single constraint—the mathematics is such that all optimized controls "see" all constraints simultaneously.

Low electrical sensitivity to one or more constraints is only part of a control's ineffectiveness. For instance, a control on its upper limit may have great sensitivity to the constraint(s) in the down-movement direction, and zero sensitivity otherwise. The effectiveness of a control depends also on its available range of motion, and this quantity is likely to vary during an OPF calculation. Effectiveness can also be assessed by the control's economic impact. Of great importance is the fact that sensitivities (particularly VQ ones) can change radically during the solution process.

In the past, certain approximate non-iterative remedial dispatch calculations used the expedient of zeroizing the small (ineffective) control-to-constraint sensitivities, as a means of preventing those controls from responding to the violations. Unfortunately, such an approach fails completely in OPF. It violates Kirchhoff's laws and as a result the successive OPF iterations lose consistency with each other.

#### *4.3.3. PVQ OPF—Defective Formulations*

In the research literature, it is not uncommon to encounter OPF problems formulated as the minimization of an objective (e.g. cost) by simultaneously optimizing P controls and VQ controls, sometimes including high-impact binary controls such as line switching and unit commitment. From a mathematical point of view, this "one-shot PVQ optimization" concept looks appealing—it promises a single unified OPF solution that seems to represent the best possible deployment of all available power system resources.

This has obvious attractions in certain cases—for instance, depending on a generator unit's capability curve, it can resolve the tradeoff between the unit's P and Q outputs to enforce network constraints. Notwithstanding, the general problem of obtaining reliable PVQ OPF solutions that meet realistic engineering criteria is still an unsolved challenge. The biggest of these challenges continues from the theme of the previous subsections. It is the fact that a mathematical optimum-seeking algorithm does not (and perhaps inherently cannot) recognize, and act accordingly, when it makes no engineering and economic sense to deploy a low-sensitivity control to enforce a constraint. Thus, the one-shot PVQ OPF formulation is highly vulnerable to producing defective, unwanted solutions. Moreover, a PVQ solution declared mathematically "optimal" can be very dangerous, because close scrutiny of it may be required before coming to the realization that it is not in fact a practical engineering solution.

To illustrate, consider economic dispatch where P controls and VQ controls are optimized at the same time. The latter controls are usually taken to be free or very cheap. Then the optimizer naturally tends to use them for constraint enforcement in preference to the higher-cost P controls, even when they have little electrical sensitivity to the relevant constraints (such as MVA or ampere flows). This easily leads to OPF solutions with impractically large VAR flows and poor voltage profiles. Note also that reactive sensitivities change radically during an OPF calculation as controls and constraints hit or back off limits.

PVQ optimization needs major breakthroughs before it can provide generally usable OPF solutions. Assigning real costs to the VQ controls, implying a simultaneous active and reactive market, is arbitrary and difficult (e.g. how do you cost transformer tapping versus shunt capacitor switching versus generator MWs?). Depending on the power system, it may be possible to identify, either by sophisticated algorithms or manually, local situations where simultaneous PVQ optimization is truly realistic and advantageous. Then extra constraints (but which and what values?), and additional logic and/or priorities, will be needed to keep VQ control local and practical.

Finally, the intended engineering use of any specific PVQ OPF formulation and solution should be clearly identified. The results might be difficult or impossible to apply. For instance, in system short-term operation, there are few means for synchronizing, sequencing or otherwise coordinating the deployments of the various disparate controls.

# 5. MODELING ASPECTS

## 5.1. Analytical Frameworks

The nodal power flow equations can be expressed in a variety of forms. There is no doubt that the unique nonlinear characteristics of each such form can affect the performance and therefore, in principle, the choice of OPF solution method. Commonly, the big choice is between the "$V\underline{/\theta}$" polar and "e+jf" rectangular coordinates for the complex voltages, and between power and current mismatches.

The pros and cons of different forms are very problem-dependent. The arguments for and against the general adoption of one form or another are unclear. For instance, with rectangular coordinates the real and imaginary current and voltage components are linearly related and it is thus claimed that this favors solution methods based on successive network-equation linearization. On the other hand, these coordinates completely sacrifice the power system’s quasi-linear P–θ relationship, and the ability to constrain voltage magnitudes directly. It appears that a polar formulation is much more widely used; this could be the efficient choice overall.

A critical, partly related, aspect of the analytical framework is the choice of decision and state variables. Consider for example the optimization of a generator's voltage subject to limits on its reactive output. In principle, the outcome should be the same if instead the reactive power were to be optimized subject to limits on the voltage. But the mathematical properties and behaviors of the two formulations are vastly different. Similar considerations apply in many other situations. For instance, do we optimize shunt reactances subject to voltage limits, or optimize their controlled voltages subject to functional limits on their reactive powers? Other examples involve transformer taps, phase shifter angles, and HVDC and FACTs variables.

### 5.2. Coupled and Decoupled Network Models

Generally, the OPF problem uses the "accurate" AC power-flow-type network model, in which all quantities are represented via the network equations as electrically coupled, i.e. sensitive to each other.

However, as previously mentioned, it is fairly common to use highly approximate[5] DC-type "model decoupled" network equations that linearly relate MW controls to MW flows. It is important to recognize that even DC-modeled OPF calculations require iterative optimization—as always, the objective, controls and/or constraints typically undergo discontinuous changes during the solution-seeking process. It seems that there are no real-life applications where a single optimization pass of a DC-modeled problem is routinely sufficient to provide usable, reliable results.

For OPF problems involving VQ controls and VQ constraints, the full AC network model is necessary. There appear to be no reliable linear VQ-only decoupled network models analogous to the DC MW-only model.

As a related comment, we should distinguish between a decoupled network model and a decoupled algorithm. Such a model is inherently approximate. The algorithm is not necessarily so. A prime example is the Fast Decoupled Power Flow, whose algorithm is decoupled but whose solution is exact because the model is coupled.

### 5.3. Internal and External Networks

In OPF calculations on a large interconnected system, control variables are nearly always designated as dispatchable only in a certain "internal" part of the network (e.g. in one company, ISO or RTO). The "external" network is modeled as operating in a power-flow-like manner. That is, it has non-optimized controls (e.g. regulating specified local target voltages) that respond to changes provoked by the optimization and the simulated contingencies.

The external network needs to be protected by individual and net (e.g. import/export) operating constraints. Otherwise, the OPF process is inherently liable to optimize the internal system at the unacceptable expense of its neighbors—by drawing heavily on external reactive resources, creating external loop flows, etc. At the same time, there is no general rule—we have also seen counter-examples where the optimization of the internal network also benefits its neighbors.

### 5.4. Control Discreteness

One of the least well-resolved aspects of OPF is the fact that many controls (whether optimized or local) operate in discrete steps. Examples are transformer taps and phase shifts. Large steps, most typically encountered with shunt and series compensators, are particularly problematic.

Usual practice is to initially treat each control as continuously variable, and perform post- and/or inter-optimization discretization on it. This produces sub-optimal solutions, with the strong likelihood that certain previously-enforced constraints may violate at the end. Elaborate heuristics can mitigate but not provide a complete cure for this. In principle, a mixed-integer programming (MIP) formulation should be able to handle all such discrete modeling. Computationally, however, this is extremely expensive, and is likely to be applied only to controls that represent very large, important discontinuities.

Discrete optimized quantities with very large step-function impacts include line switching in/out, bus splitting/merging, load shedding and generator start-up/shutdown. Among the candidate applications are post-contingency corrective OPF (which might include deliberate system separation), restorative control, and transmission planning.

In a non-small power system, it is unlikely that the optimization of network topology (and all the other binary variables) will ever be attempted in a very general manner. Otherwise, the computational problem would unnecessarily become enormous. Instead, engineering pre-analysis and experience with the power system can identify most scenarios where switching in/out is potentially effective.

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[5] B. Stott, J. Jardim, O. Alsaç, "DC power flow revisited", IEEE Trans. Power Syst., Aug. 2009.

In fact, if the number of binary combinations is limited and localized, the use of solve-and-compare sub-optimal solution-seeking techniques (already practically implemented) are likely to be much more efficient than MIP formulations.

### 5.5. Equality Constraints

The principal equality constraints in the OPF problem are the power flow equations, whose overall nonlinearities seem to depend mostly on the power factors and angles of transmission and on the R/X ratios of the branches. These equations may include loads (and corona losses) as functions of voltage.

Depending on the formulation, other equality constraints specify the targets for local controls. Examples are the voltages controlled locally or remotely by reactive sources such as generators, transformers and shunts, as well as the relatively complicated models for two and multi-terminal HVDC links and FACTs devices. Whenever a local control hits or backs off its limit, the network equality constraints and sensitivities change abruptly. Two other examples are the following:

- Characteristics such as transformer impedance variation with tap and phase shift are usually given as tabular data. For optimization purposes, they might be curve-fitted as model equalities. In practice, this can complicate convergence—for instance, phase shifter impedance can change very nonlinearly and approach zero. Suboptimal approaches involving successive impedance adjustments during the solution are more common.
- Loads may have discontinuous model transition points at sufficiently low or high voltages.

The sharing formulas (as opposed to priorities) between optimized or local controls are also usually expressed as equalities, which change abruptly when limits are hit or backed off. Examples are:

- Generator VARs within a plant.
- Taps or phase shifts of parallel transformers.
- Taps of generator step-up transformers.
- Shunts on the same or electrically adjacent buses.
- Distributed slack MW power.

### 5.6. Inequality Constraints

A large security-constrained OPF problem involves a vast number of inequality constraints, and the only way to handle them is to conduct intelligent searches outside the central optimizer for the critical set of contingencies that will include all eventually-binding constraints. Considerable heuristic logic for screening and filtering is needed, and this is among the most important parts of the entire OPF solution process. The critical set is updated at each "outer loop" OPF cycle. The current critical set is processed within an "inner loop" cycle. Many specific variants on this process, including intermediate loops, are possible.

The vast majority of inequality constraints are expressed as fixed upper and/or lower limits on problem variables or on continuous functions of those variables. In principle, these are readily fed in exact or approximated (e.g. linearized) form into the central constrained optimizer. However, some limits are too complicated and/or computationally expensive to handle this way. They have to be successively updated in between optimization iterations, introducing discrete perturbations (discontinuities) into the problem. An example is the voltage-varying capability curve of a synchronous generator. A particularly difficult case (in most OPF approaches) is a limit on the pre-to-post-contingency change in any quantity. Examples are the voltage change at a consumer point and the change in a generator MW output (i.e. a ramping rate constraint).

Even more complicated is the type of constraint where certain prescribed actions take place once a trigger value is exceeded in a pre or post contingency state. This includes relay actions, switching, generation shifting, load shedding or even unit commitment, to simulate automatic control responses or operator remedial procedures. The problem definition has to specify whether or not the OPF calculation must prohibit the trigger value from being exceeded (expensive but analytically simpler) or can accommodate the action (potentially more economical).

### 5.7. Locally-Acting Controls

Locally-acting controls represent the power system's automatic response to system disturbances and changes in the optimized quantities. As indicated in Subsection 5.3 above, they are virtually always modeled as such in the external network (if any). They can include the responses of generator MWs and voltage regulators, in-phase and out-of-phase transformer taps, shunt and series reactive compensation, FACTS devices, HVDC converters, and so on.

Of great importance, it should be noted that local controls also need to be modeled in the internal network. Per Section 4.3.3, it is rarely permissible to optimize active and reactive controls simultaneously. Thus, for example, when internal-network P controls are optimized, all VQ controls (internal and external) are generally represented in locally-acting (i.e. power-flow) manner.

This is a major source of complication for OPF formulations and solutions. Most local controls can be expressed by one or more equations. However, throughout the iterative OPF solution, each of them can hit or back off a limit, and sometimes they do this in a repetitive, oscillatory manner. Each such transition involves a control-mode switch-over, and the substitution of one model equation with another. The resulting abrupt change in network sensitivities can be very severe—something that is difficult or impossible to handle inside the central optimizer. A prime example is the familiar switching between power flow PV and PQ bus types. Another is the highly discontinuous response of limit-enforced generators to system MW imbalances due to the outage of generation and load and/or to network islanding, as mentioned previously.

Thus, local control models have a major influence on the speed, path-dependency and reliability of the OPF optimum-seeking process, and on the non-uniqueness of the eventual solution.

Attempts to work around the troublesome effects of local controls are sometimes made by designating these controls as optimized (within their limits), together with appropriate local target constraints and costs. Such attempts create their own analytical and algorithmic problems. In particular, the validity of the resulting OPF problem and its solutions is in question. For instance, how to realistically represent post-contingency responses (e.g. PV-to-PQ bus mode switching), and how to assign costs to the controls (e.g. should they be encouraged to move, or not to move, and by how much)?

Like optimized controls, the local controls, their variables and their constraints can be expressed in different ways that strongly affect the mathematical characteristics of the problem.

### 5.8. Constraint Hardness/Softness

Some power system constraints have fixed, "hard" equipment-based limits (e.g. transformer tap and switchable shunt ranges). However, most OPF operational constraint limits are fuzzy. Clearly, for instance, a 500 MVA transmission line flow limit is entirely nominal. Yet most optimizers will treat this limit as "hard". We have to ask: when this constraint is a bottleneck in the OPF calculation, what would be the physical and economic consequences of allowing it to go "soft" (relaxed, expanded), and to what degree? This is a well-known often-posed OPF question which, considering its importance, has received insufficient practical attention.

The shadow price of a binding constraint during or after an OPF solution certainly provides quantitative feedback about the desirability of expanding the limit. But then, how much should that limit be expanded, before re-running the OPF calculation? This requires well-defined, experience-based rules. What is the economic penalty for such limit relaxation?

The above questions are somewhat rhetorical, because an OPF solution can have many simultaneously interacting binding constraints. In general, therefore, it becomes extremely difficult to decide which ones to relax, and by how much. Cycling the OPF calculation to a sensible successive-relaxation outcome using some appropriate logic is highly problematic. Worse, the OPF problem might actually be infeasible, in which case any approach based on binding constraint shadow prices does not work.

An alternative is to model each operating (but not equipment) constraint as inherently "soft" or "elastic", with a monetized progressive (e.g. quadratic) penalty cost for violation of its nominal limit. A very suitable type of penalty enforces the limit as hard up to a threshold marginal cost, and becomes progressively very steep as the violations increase. In this way, soft constraints tend to share the violations among themselves, tending not to over-stress any one point of power system operation to the point of solution divergence.

Few publications have addressed the monetization of constraint expansion in specific applications (e.g. security constrained economic dispatch). Much more work is needed.

### 5.9. Modeling Consistency

Computational economy is sometimes sought by using a mix of network models—for example, accurate AC for the pre-contingency network and (approximated) incremental DC for post-contingency MW-flow violations. Such approaches involve big pitfalls. Great care should be taken to avoid model inconsistencies, which will manifest themselves at each cycle of the OPF process. When the models do not match, it very easily happens that a contingency constraint enforced in the central optimizer is not even close to its limit during the following contingency analysis and monitoring pass. This creates big problems in obtaining convergence and declaring that a satisfactory solution has been reached. A similar phenomenon occurs when small control-to-constraint sensitivities are neglected.

## 6. SOLUTION-SEEKING ISSUES

### 6.1. Optimality

The goal of a typical OPF solution is to satisfy the power system physical and operational constraints, scheduling the control variables to minimize (or maximize) a scalar objective function. Generally, satisfying the constraints and deploying the controls in practically-achievable manners takes precedence over achieving the absolute highest degree of optimality.

One of the recurrent messages of this paper is that the models, and possibly the controls and objectives, change repeatedly during a typical practical OPF solution process. So, in any non-trivial OPF problem, the concept of absolute optimality (global or local) is elusive, even to the point of becoming a spurious issue.

The optimization problems of all complex industrial systems have the same nature—they are too complicated to express (or even approximate) in any smooth, piecewise-smooth, and/or binary mathematical forms. Their "optimal" solutions virtually always have some degree of solution path dependency. The best that can be achieved is an optimum of the very last problem approximation. In reality, "optimal" means "very good".

Having said this, there are specific, usually market-related, applications where OPF solution repeatability is regarded as important, and this implies obtaining a true and possibly unique optimal solution point. Unfortunately, repeatability (e.g. getting identical solutions from different starting points and/or with different software) is a goal that might only be achievable, if at all, with the most trivial, simplified, linearized models and formulations.

### 6.2. Priorities, Rules, Infeasibility

More than anything, the merit of an OPF program is to be judged by its ability to produce useful solutions under onerous operating conditions, where the integrity of the power system is at risk. Infeasibility—the inability to satisfy all constraints—is clearly a major risk indicator.

In the very common event of infeasibility, any OPF algorithm should identify the points of weakness, so that the user can check engineering plausibility and veracity, and investigate potential data and model problems.

As regards optimized controls, in a text-book OPF problem they are all defined at the beginning, and the solution ends up being pronounced feasible or not. Commonly, however, preferences for optimized controls are specified in the form of priorities. These can work in different ways. For example, only the high-priority controls are first optimized. If the problem is infeasible, the next-highest priority controls are added, and the solution is repeated. Exactly the opposite approach is also valid: start with all controls and subtract lower-priority ones until infeasibility is reached.

Constraint elasticity as described in Subsection 5.8 above can be extremely valuable in detecting and identifying infeasibility, and in arriving at least-infeasible solutions. This can work in conjunction with corresponding constraint-relaxation priorities, involving discrete expansions or contractions of limits according to feasibility.

One specific issue is the constraint that turns out to be unenforceable in multiple contingency cases. Is the constraint to be relaxed in each such case, how, and by how much?

### 6.3. Overall Convergence Criteria

The central optimizer, which at each iteration handles an approximation to the OPF problem in some standard mathematical form, has a stopping mechanism based on perceived optimality, infeasibility, etc. of the approximated problem, often with quantitative indices.

Since the OPF calculation involves one or more model-solving and redefining loops outside the central optimizer, overall convergence is recognized when at least three criteria are met: (a) the most recent problem approximation has solved optimally, (b) all equality and inequality constraints are satisfied, and (c) continued iteration produces no significant changes. As previously mentioned, the complexities and discontinuities of the OPF problem suggest that there are no useful analytical convergence or optimization conditions for these types of problems.

### 6.4. Convergence Difficulties

OPF calculation convergence difficulties can occur for many reasons, including high OPF problem nonlinearity, genuine static instability of the modeled power system, and insufficiently powerful central optimization. Perhaps the biggest single culprit is poor system modeling and data errors. Model discontinuities and degeneracy also seem to be high on the list.

Powerful mechanisms for overcoming both small and large oscillations at different stages of an OPF calculation are needed in any practical code. These techniques have major heuristic components and they require considerable experience and experimentation. They can include damping, backtracking, freezing and tie-breaking, and have path-dependency effects on the eventual solution.

The most problematic and least-addressed phenomenon is when either the pre- or (usually) the post-contingency power system operating state fails to converge. This can be due to algorithmic inadequacy or the fact that the current system state has been driven into (or started in) an unstable operating condition. One unsatisfactory practice is for the algorithm to report and discard any unconverged contingency case, which leaves the eventual solution unsecured for that contingency. Much more work on this is required.

### 6.5. Degeneracy

Many OPF problems exhibit degeneracy, which is when an “optimal” objective function value can be achieved by an infinitive number of solutions (this is distinct from multiple local optima).

One consequence of degeneracy and near-degeneracy is cycling in the central optimizer and/or in the outer-loop OPF iterations, long solution times, or even outright convergence failure. Another consequence is arbitrary OPF solutions, which are particularly undesirable in a market-oriented dispatch or auction that demands equitable, auditable awards and marginal costs.

It is difficult to eliminate a-priori (i.e. by pre-processing the data) all tendencies of a particular OPF problem towards degeneracy. Likewise, it is difficult to post-process many such situations. To cater for all such situations, the OPF solution process should internally detect, resolve and report degeneracy and near-degeneracy automatically, in a manner suitable for system operation/markets.

OPF degeneracy is normally associated with certain controls and/or constraints. However, these are often not easy to identify. Also, each central optimizer deals (if at all) with degeneracy in different ways—it may arbitrarily select one of the solutions, or it may apply internally-coded tie-breaking rules that are rarely what the engineer or market user wants. In many cases, the optimizer may give no indication that, and how and why, the problem is degenerate.

In the context of OPF, the two most common types of degeneracy are outlined here:

#### *6.5.1. Control Degeneracy*

In operations research terminology, this is a form of dual degeneracy, and it occurs most frequently in OPF when two or more optimized controls with linear cost curves are equally eligible to be marginal and have the same electrical sensitivities to the binding network constraints. Classic examples are the many closely-coupled VQ controls whose mutual sharing or priority characteristics are typically never specified in the problem formulation. In MW dispatch, the simplest example is that of identical generating units on the same bus, or in the same plant, or in some other symmetrical arrangement. Financial transmission rights allocations and auctions are particularly vulnerable to control degeneracy. Some degenerate control cases are easy to identify a priori, but many are not.

#### *6.5.2. Constraint Degeneracy*

This corresponding primal degeneracy condition is encountered when non-binding constraints are on their limits at the solution. Any one of these non-binding constraints could have been designated as binding without affecting the deployment of controls.

One simple example is a set of branches in series, with the same rating and carrying the same flow—each branch is an equal candidate for becoming binding. A corresponding example is that of a set of identical branches in parallel. These constraints are redundant—they are effectively identical.

However, unlike these trivial topologies, other cases are difficult to identify, and the seemingly-arbitrary choice between constraints that are equally eligible to be binding can have a big effect on the values of the prices. Specific market participants may end up paying too much or too little for their transmission access or hedging.

Degeneracy also frequently occurs when a constraint is binding in one contingency case and is on its limit in other cases. It also occurs in time-linked OPF problems, when a constraint is binding in one time period and on its limit in other periods. Again, this can materially affect market prices.

### 6.6. Constraint Insensitivity

In an OPF solution, the mathematical optimizer has the task of enforcing all constraints using the available control (decision) variables. When a constraint is difficult to enforce, the optimizer may cause large moves in controls (from their initial or otherwise values) that have little effect on the constraint. The optimizer is performing its function correctly but, being unable to perceive that this makes no engineering sense, it can produce an impractical OPF solution. Moreover, the causes of the problem may be obscure.

This is a serious trap that must be avoided during most types of OPF solutions via special sensitivity calculations and techniques of varying sophistication. At minimum, a constraint with little sensitivity to any control should be identified. Then the constraint can automatically be ignored, or extra control activation can be triggered on a priority-rule or price-threshold basis. Such identification must be carried out repetitively, since sensitivities can change radically during the solution process, noting also that controls already on limits have completely different sensitivities in the up/down directions. OPF calculations without such safeguards are inherently liable to give unreliable results.

### 6.7. Marginal Costs

An OPF solution is generally expected to include marginal costs such as the shadow prices of congestive (binding) constraint. In some modern applications, it is important to obtain the prices of nodal injections, particularly in markets based on LMP (Locational Marginal Prices). A complication is that these prices are liable to be "contaminated" by extraneous objective-function components such as those described in Section 4.2. Obtaining LMPs that are "clean enough" for market transparency can be a challenge, much more with some optimization methods than others.

In addition, a frequent requirement of an OPF solution is the decomposition of marginal costs into their components—for example, the energy, loss and congestion components of an LMP, and shadow price breakdowns. Again, this is straightforward with some optimization approaches such as LP, and difficult with others. The choice of system reference is well known to affect the components, and removing this arbitrariness requires the rigorous incorporation of a distributed reference into the OPF formulation and solution process.

# 7. NOTES ON OPTIMIZATION

## 7.1. General

Obviously, there is no "one size fits all" best optimization approach for the range of problems that come under the umbrella of OPF. The choice of approach is heavily influenced by the types of OPF problems to be addressed.

Power system modeling and practical solution-seeking mechanisms are sufficiently complex, and many networks are so big, that large sub-processes of an OPF calculation have to be performed outside the core optimizer. These sub-processes cannot possibly be handled with adequate modeling, efficiency and reliability inside the general-purpose solvers. They need well-developed, well-tried power-system-specific methods and code. Examples are contingency analysis, the identification and filtering of critical constraints and contingency cases, and in most cases the computation of sensitivities. Attempts to embed these calculations within the core optimization might work, if at all, only for very small networks and very simplified models.

## 7.2. Commercial Optimizers

Commercial general-purpose optimizer packages are becoming more and more powerful, and are increasingly attractive as core elements of any OPF solution process. Some of them can address aspects of OPF—such as binary decision variables—that would otherwise be very difficult. Some of them are good at handling nonlinear constraints and non-convex objectives. In many cases, a multi-algorithm optimization package can be of great advantage for efficiency and reliability—for example, it can start by solving more approximate objectives and constraints (e.g. linear or piecewise linear) and then switch over to more accurate ones later in the process.

Clearly, no general-purpose optimizer can handle a big contingency-constrained OPF problem "as is"—the problem would be immense. Instead, as illustrated in Figure 1 of Section 2.3, the optimizer becomes the core of an iterative process, fed by successive approximations to the objective function and the pre- and post-contingency constraints. There are significant modeling discontinuities in between iterations. Certain of these discontinuities might be expensively handled by a mixed integer formulation, but this would not be practical or even possible for the vast majority of them.

As previously indicated, this means that only a fraction of the OPF calculation can take place inside the central optimizer—most of the calculation must take place outside it. This is not just a function of computing power. It will remain true however fast the hardware becomes.

In general, the solution methodology has to be matched with the OPF problems to be addressed. The specific candidate software (within and outside the central optimizer) has to be evaluated. The exploitation of parallel computation must be maximized.

These outcomes are influenced enormously by implementation factors that include the computing hardware, operating systems, compilers (and their optimizing parameters), and multi-core exploitation such as multi-threading and MPI.

## 7.3. Hot/Warm Start

One highly desirable property of an OPF optimizer is its hot start capability. A typical OPF solution process has to cycle through the central optimizer many times, each time updating the model approximations, the critical constraints/contingency cases, and the solution-seeking rules that may change the objective function definition. In this way, the optimizer receives successively refreshed problems to solve, each of which has changed incrementally but often not smoothly from the previous cycle. In performance-critical applications, it is important for the optimizer to take good advantage of the results from the previous cycle, i.e. have good hot start capability.

An optimizing package's hot start efficiency may not be self-evident, and may need close experimental scrutiny. For instance, it turns out that some LP-based codes have extremely efficient hot starts, while some Interior Point programs have no hot-start capability because at each pass they revert to an interior point.

# 8. OTHER ISSUES

## 8.1. Extended OPF Formulations

At the research level, OPF problems are being formulated that address the optimization of power system operation more comprehensively. Examples are the inclusion of: topology optimization, stochastic objectives and constraints, fuzzy optimization, formal variable discreteness treatment, stability constraints, multiple objectives, time-varying optimization with temporal constraints, and (see Section 4.3.3) the simultaneous optimization of all available controls.

The more ambitious the OPF formulation, the more faith it implies in the power of modern optimization and computer technology to solve increasingly huge, complicated electric power system problems. It is not at all clear how much of this faith is well founded, or which of the proposed extended functionalities address practical major engineering issues. Possibly, some of this trend might reflect the oft-quoted strategy: “if a problem proves sufficiently difficult, expand its scope”.

In any case, such problem extensions are largely outside the range of the present paper, which is oriented towards the more fundamental solution requirements that apply throughout the various OPF applications.

## 8.2. Computation, Software, Education

It is axiomatic that the most important problems formulated by the industry require the next generation or two of computers. This is certainly true in the OPF field, where there is relentless pressure to solve increasingly advanced OPF problems faster. Such challenges are most onerous in power system operations and control—reliable solutions with accurate, nonlinear, discontinuous modeling have to be computed within short elapsed times.

Practical OPF development obviously involves good design and implementation for software, algorithms, modeling and user interfaces. A good balance must be achieved between software efficiency, maintainability and extendibility. OPF developers thus require a combination of power engineering, mathematics and IT skills. Today, such developers are in very high demand. However, they are in very short supply, which is surprising, considering that graduate electric power systems study has now become very popular again around the world.

The industry is attributing this shortage to the trend in power system graduate education to de-emphasize detailed computation and numerical analysis. Most academic research has become dependent on very powerful general-purpose "black box" prototyping tools that accelerate research results and technical publication. These tools allow students to perform seemingly-large calculations without gaining the insight, understanding or skills that are needed for complex industrial-quality analysis.

## 8.3. Parallel Computing

The scope for using multiple processors to handle the number-crunching requirements of modern OPF formulations is a subject of great interest and some frustration. During "outer-loop" searches for critical contingency cases and constraints, it is possible to distribute the solutions of the cases between different processors.

The most challenging bottlenecks are in today’s central constrained optimization algorithms and codes where, because of serial logic, speed-ups of a few times at most are possible utilizing multiple CPUs. This is particularly true for the modern codes that take full advantage of problem sparsity.

Radically new optimization algorithms and codes will have to be developed before high-performance computing with multiple CPUs (and possibly graphic processor units) can deliver orders of magnitude speed-ups.

## 8.4. Re-dispatch Sequencing

In power system operation and control, an important question is how to sequence the dispatching of the controls between their initial and OPF-solved values. This is one more issue completely absent from the classical OPF problem formulation. Different controls can be moved at different rates and with different dead bands. This issue is particularly pronounced with the problematic PVQ optimization of Section 4.3.3, particularly with line switching (including restorative control). Clearly, inappropriate control-change trajectories can compromise system security and to some extent economy. Preset control priorities are unlikely to cover all cases. For such applications, control sequencing should in principle be an integral part of the relevant OPF problem formulations and solutions. Sophisticated post-processing of the OPF solution might provide a partial answer. This is a difficult problem on which not enough work has been done.

# 9. CONCLUDING REMARKS

OPF represents a class of problems with a very wide and increasing range of critical power system applications. OPF has already become an important component of many operational, planning and market processes. Nevertheless, after fifty years, the field is still very much a work in progress and there is huge scope for further development.

It seems rather misleading to distinguish between "OPF" and "SCOPF", since optimization without security (contingency) constraints has limited uses. At the same time, it is noted that the definition of security itself is bound to change over time, together with the ways in which it is handled in OPF applications.

Approaching any OPF formulation and calculation as a clean mathematical problem does not seem constructive, even as a first-stage simplification. Over the years, this has led to a huge amount of work that is not adaptable to the much more complicated problems encountered in real-life OPF.

Several general trends in the field are noted:

- Modern OPF formulations increasingly address more complex, comprehensive problems, and OPF solutions are increasingly embedded in bigger calculations.
- There is more widespread use of powerful general-purpose commercial packages as the "central optimizer". The optimizer has to be chosen very carefully for its properties vis-a-vis the OPF problems being addressed.
- The overall OPF solution process involves considerable calculation, modeling and solution-seeking control outside, and iterated with, the central optimizer. These "outer" processes have a major (typically the major) influence on a successful OPF outcome.
- The closer to real-time operation, the more important power system modeling accuracy tends to be. The more detailed the modeling, the more difficult, discontinuous and path-dependent the solution process is likely to be, and this has a big impact on the efficiencies of the powerful general-purposes optimizers.

For any power systems application requiring OPF, a thorough understanding of the most important factors seems essential. These factors can then be evaluated against the characteristics of the OPF methods and tools that are currently available, or might be developed. In this respect, it is hoped that the comments in this paper can be helpful.

# 10. ACKNOWLEDGEMENTS

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# 11. SUBJECT REVIEWS

This paper makes no attempt to provide comprehensive OPF references. The reviews below contain extensive bibliographies on the subject.

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