



Upravljanje robotskim sustavima III ciklus predavanja

Zračna robotika: Беспilotne letjelice (UAVs)

Upravljanje formacijama

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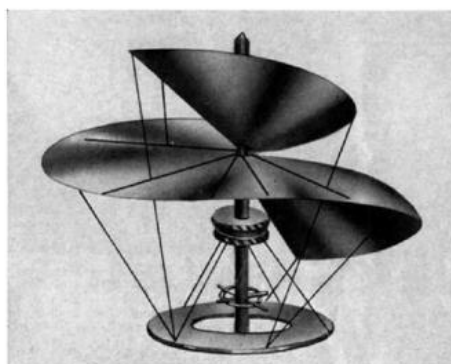
Sadržaj

- Povijest i trenutno stanje
- Matematički modeli letjelica
- Upravljanje letjelicama
- Upravljanje formacijama



Povijest i trenutno stanje

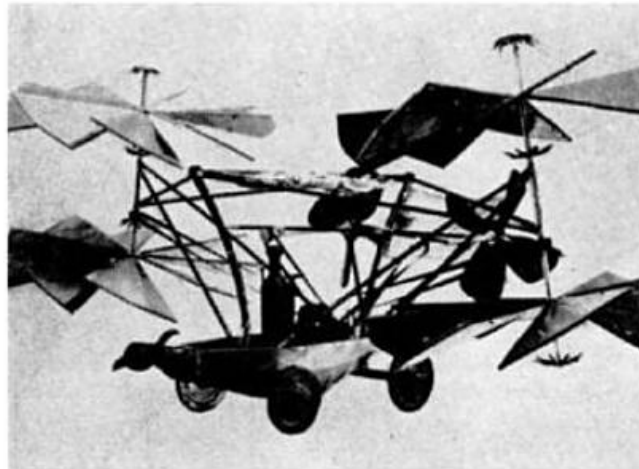
- 425 BC – Archytas => mehanička ptica
- 1483 – da Vinci => zračni vijak





Povijest i trenutno stanje

- 1843 – George Cayley => zračna kočija





Povijest i trenutno stanje

- 1907 – Paul Cornu => helikopter



“Like all novices we began with the helicopter but soon saw that it had no future and dropped it. The helicopter does with great labor only what the balloon does without labor....The helicopter is much easier to design than the airplane but it is worthless when done.”
Wilbur Wright, 1906



Povijest i trenutno stanje

- 1909 – Igor Sikorsky => bespilotni helikopter (vibracije)
- 1912 – Boris Yurev => repni rotor

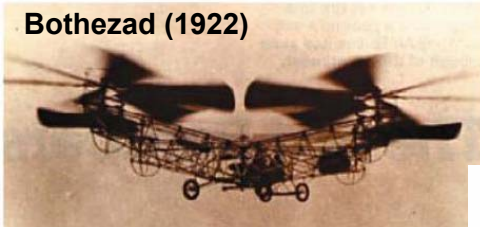




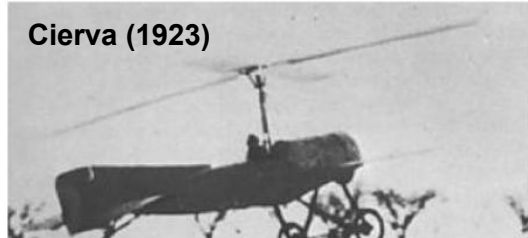
Povijest i trenutno stanje

Post-WWI design

Bothezad (1922)



Cierva (1923)



Fa-61 (1936)



Sikorsky (1939)





Povijest i trenutno stanje

Moderno doba **MQ-1C Warrior (2008)**



MQ-9 Reaper (2007)



RQ-8A Fire Scout (2002)



Schiebel CAMCOPTER (2005)



Povijest i trenutno stanje

Zračna robotika - aplikacije

Vojne

- Target and decoy – mete za vježbanje gađanja protuzračne obrane
- Praćenje i izviđanje – informacije o bojnopolju
- Sudjelovanje u borbi – napadačke sposobnosti u riskantnim misijama
- Logistika – posebne bespilotne letjelice za prijenos tereta



Povijest i trenutno stanje

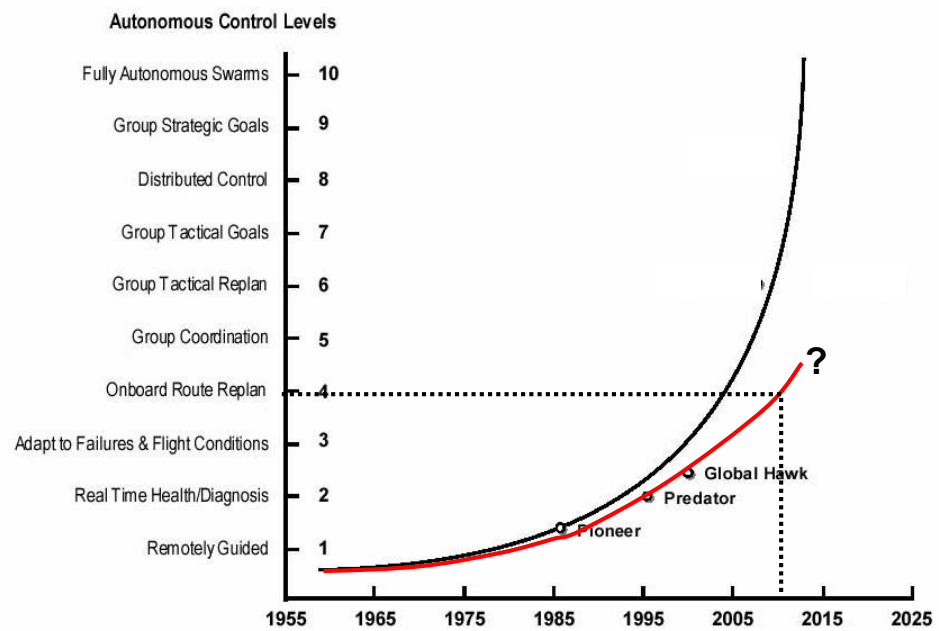
Zračna robotika - aplikacije

Civilne

- Nadgledanje prometa i vremena
- Vatrogastvo
- Poljoprivreda
- Traženje&spašavanje
- Istraživanje i razvoj

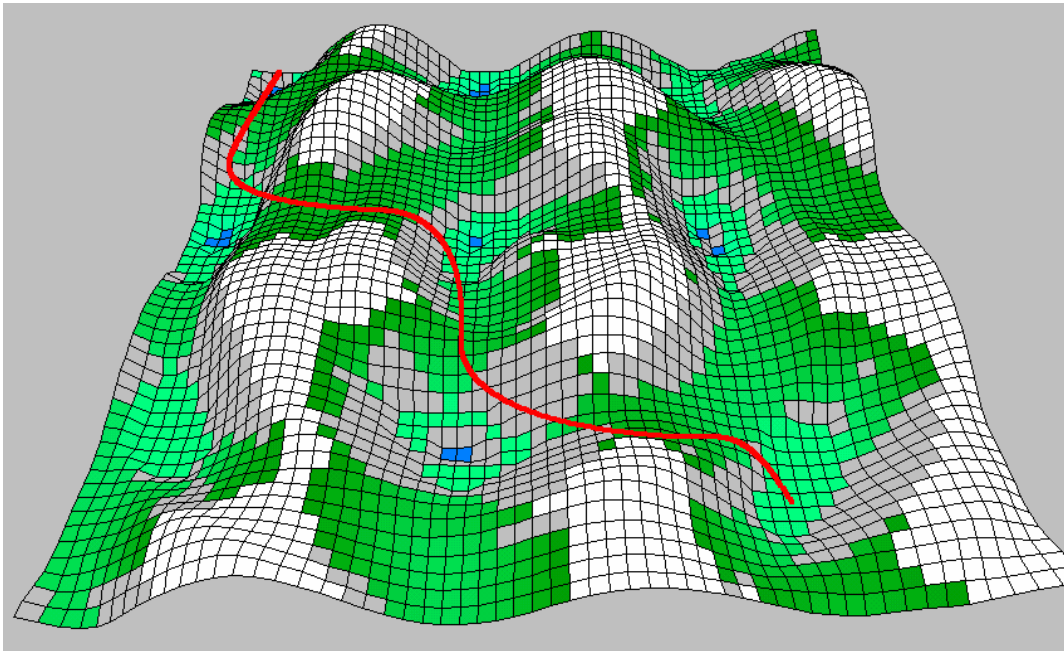


Povijest i trenutno stanje



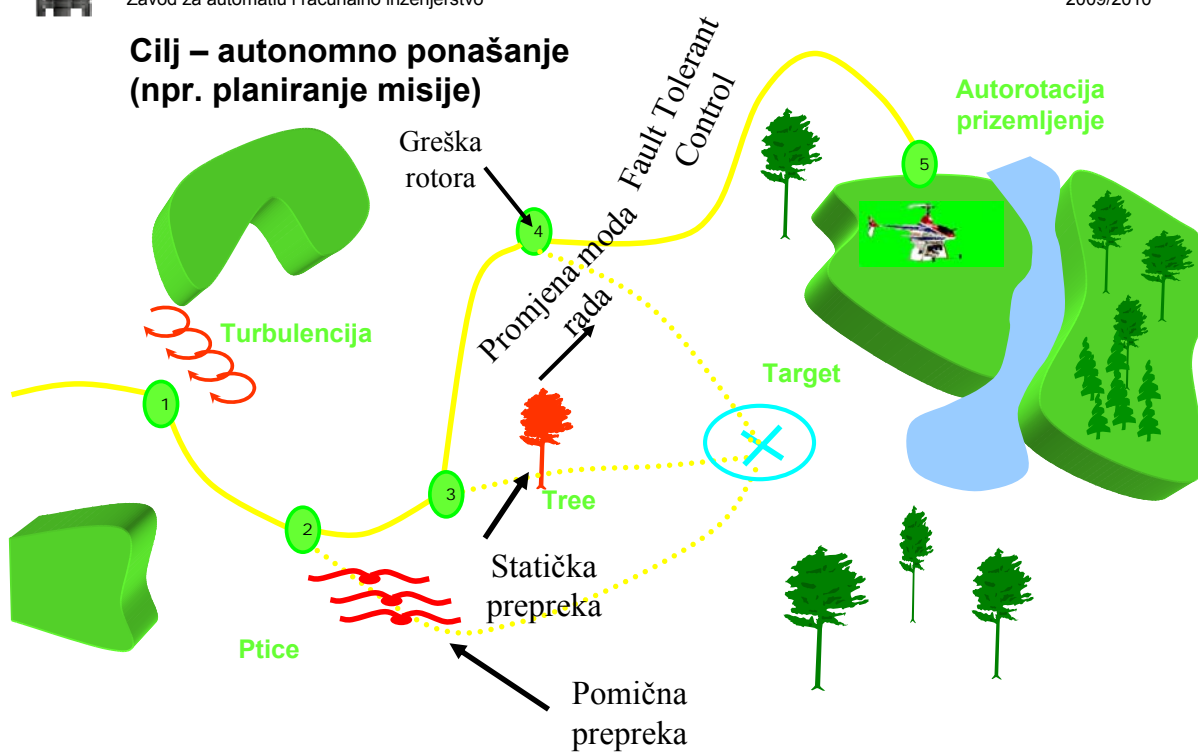


Cilj – autonomno ponašanje (npr. planiranje trajektorije)





Cilj – autonomno ponašanje (npr. planiranje misije)





Problemi na kojima se radi (*Paul Y. Oh*)

Vehicle performance difficult to acquire, assess and validate because flight conditions are often dynamic and unpredictable (e.g. wind and lighting). Absence of data has led to ad hoc design. Consequently experiments performed by other institutions fail to yield similar results.

Creating sensors and algorithms for autonomous collision avoidance and localization without GPS is a challenge. If overcome, flying in near-Earth environments (forests, tunnels, and inside buildings) could be realized..

Making propulsion systems that are light-weight, low-noise and provide longer flight times is an engineering challenge.

Communications that is robust, secure and unjammable is an unsolved problem. Improved this technology would vertically impact the development of distributed and multi-vehicle systems (e.g. swarms).



Matematički modeli letjelica

6 stupnjeva slobode (6 DOF) – x , y , z , *valjanje*, *poniranje*,
skretanje

UAV kao sustav je:

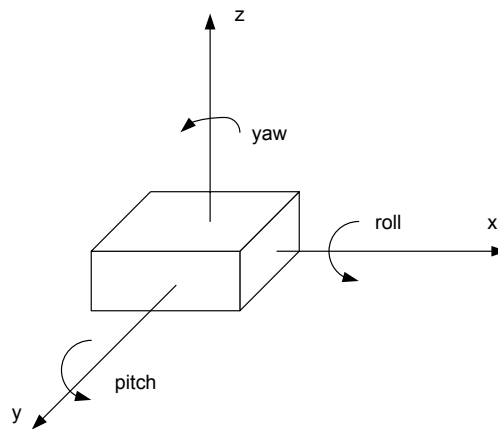
Nelinearan

Multivarijabilan

Spregnut

Podaktuiran

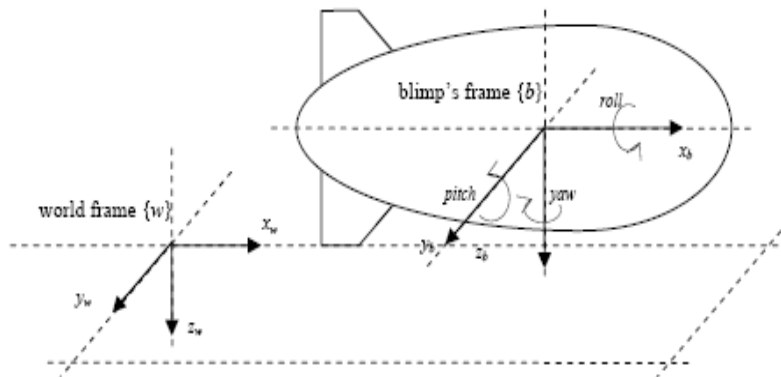
=> *Složen za upravljanje*





Matematički modeli letjelica - balon

Envelopa punjena helijem – negativan (ili neutralan) uzgon



nepomični koord. sus. \Leftrightarrow koord. sus. balona
(matrice transformacija)



gondola s tri aktuatora (propeleri)



Kinematički model balona

$$\boldsymbol{\eta}_z = [x \quad y \quad z \quad \phi \quad \theta \quad \psi]^T$$

$$\mathbf{v}_b = [v_x \quad v_y \quad v_z \quad \omega_x \quad \omega_y \quad \omega_z]^T$$

$\boldsymbol{\eta}_z$ - vektor pozicije i orijentacije balona s obzirom na koordinatni sustav zemlje,

\mathbf{v}_b - vektor linijskih i kutnih brzina balona u vlastitom koordinatnom sustavu.

Transformacija koordinata iz pomičnog u nepomični koordinatni sustav.



Kinematički model balona

Matrice rotacije

$$\mathbf{R}_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \quad \mathbf{R}_y = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \quad \mathbf{R}_z = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Množenjem matrica \mathbf{R}_x , \mathbf{R}_y i \mathbf{R}_z dobiva se

$$\mathbf{J}_1 = \begin{bmatrix} \cos \psi \cos \theta & -\sin \psi \cos \phi + \sin \phi \sin \theta \cos \psi & \sin \phi \sin \psi + \cos \phi \sin \theta \cos \psi \\ \cos \theta \sin \psi & \cos \psi \cos \phi + \sin \psi \sin \theta \sin \phi & -\cos \psi \sin \theta + \sin \psi \sin \theta \cos \phi \\ -\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi \end{bmatrix}$$



Kinematički model balona

Za kutne brzine vrijedi

$$\mathbf{J}_2 = \begin{bmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi / \cos \theta & \cos \phi / \cos \theta \end{bmatrix}$$

što konačno daje

$$\dot{\mathbf{q}}_z = \begin{bmatrix} \mathbf{J}_1 & [\mathbf{0}]_{3 \times 3} \\ [\mathbf{0}]_{3 \times 3} & \mathbf{J}_2 \end{bmatrix} \cdot \mathbf{v}_b$$

= > transformacija iz pomičnog koordinatnog sustava balona
u nepomični koordinatni sustav zemlje



Dinamički model balona

$$\mathbf{M} \cdot \dot{\mathbf{v}}_b + \mathbf{D}(\mathbf{v}_b) \cdot \mathbf{v}_b + \mathbf{g}(\boldsymbol{\eta}_b) = \boldsymbol{\tau}_b$$

$\mathbf{M} = \mathbf{M}_{RB} + \mathbf{M}_A$ - matrica masa (masa krutog tijela i dodane mase),

$\mathbf{D}(\mathbf{v}_b)$ - otpor zraka po pojedinim komponentama kretanja,

$\mathbf{g}(\boldsymbol{\eta}_b)$ - vektor djelovanja gravitacijske sile i uzgona,

$\boldsymbol{\tau}_b$ - vektor sila i momenata.

Coriolisova i centripetalna sila zanemarene



Dinamički model balona

$$\mathbf{M}_{RB} = \begin{bmatrix} m[\mathbf{I}]_{3 \times 3} & -m\mathbf{S} \\ m\mathbf{S} & \mathbf{I}_b \end{bmatrix}$$

- matrica masa krutog tijela.

$$\mathbf{S} = \begin{bmatrix} 0 & -z_{cg} & y_{cg} \\ z_{cg} & 0 & -x_{cg} \\ -y_{cg} & x_{cg} & 0 \end{bmatrix}$$

$$\begin{bmatrix} x_{cg} & y_{cg} & z_{cg} \end{bmatrix}$$

centar mase u koor. sus. balona

za balon elipsoidnog oblika (glavna poluos a , sporedna poluos b) vrijedi

$$\mathbf{I}_b = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \quad \begin{aligned} I_{yy} = I_{zz} &= \frac{4}{15} \pi \rho a b^2 (a^2 + b^2) \\ I_{xx} &= \frac{8}{15} \pi \rho a b^4 \end{aligned}$$



Dinamički model balona

Tijelo koje ubrzava ili usporava, mora istisnuti dio volumena fluida koji ga okružuje – “dodana masa”.

$$\mathbf{F} = m \cdot \mathbf{a} \Rightarrow \mathbf{F} = (m + m_{add}) \cdot \mathbf{a}$$

Za kruto tijelo dodana masa postaje tenzor sa komponentama koje ovise o smjeru gibanja tijela.

$$\mathbf{M}_A = \text{diag} \{ a_{11}, a_{22}, a_{33}, a_{44}, a_{55}, a_{66} \} \quad \text{- dodana masa.}$$



Dinamički model balona

Za tijelo elipsoidnog oblika:

$$a_{11} = \frac{\alpha_0}{2 - \alpha_0} \cdot m$$

$$a_{22} = a_{33} = \frac{\beta_0}{2 - \beta_0} \cdot m$$

$$a_{44} = 0$$

$$a_{55} = a_{66} = \frac{1}{5} \cdot \frac{(b^2 - a^2)^2 (\alpha_0 - \beta_0)}{2(b^2 - a^2) + (b^2 + a^2)(\beta_0 - \alpha_0)} \cdot m$$

$$\alpha_0 = \frac{2(1 - e^2)}{e^3} \left(\frac{1}{2} \ln \left(\frac{1 + e}{1 - e} \right) - e \right)$$

$$\beta_0 = \frac{1}{e^2} - \frac{1 - e^2}{2e^3} \ln \left(\frac{1 + e}{1 - e} \right)$$

$$e = 1 - \left(\frac{b}{a} \right)^2 \quad m = \frac{4}{3} \pi \rho a b^2$$



Dinamički model balona

Matrica **D** - otpor zraka po pojedinim komponentama kretanja.

Zbog simetričnog oblika balona **D** je dijagonalna matrica:

$$\mathbf{D} = \text{diag} \left\{ D_{v_x v_x} \cdot |v_x|, D_{v_y v_y} \cdot |v_y|, D_{v_z v_z} \cdot |v_z|, D_{w_x w_x} \cdot |w_x|, D_{w_y w_y} \cdot |w_y|, D_{w_z w_z} \cdot |w_z| \right\}$$

$$D = C_{tr} \cdot S \cdot \frac{\rho}{2} \quad C_{tr} = 0.1-0.4 \text{ za sferne oblike}$$

S - površina presjeka tijela (okomito na pravac gibanja) $S_{v_x v_x} = b^2 \cdot \pi$

$$S_{v_z v_z} = S_{w_z w_z} = a \cdot b \cdot \pi$$

$$D_{v_y v_y} = D_{w_x w_x} = D_{w_y w_y} = 0$$



Dinamički model balona

- vektor djelovanja gravitacijske sile i uzgona
(pretpostavka da balon lebdi kada na njega ne djeluju sile)

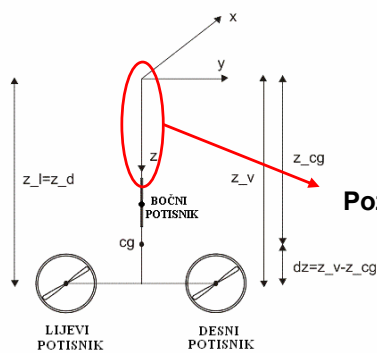
$$\text{uz } cg = \begin{bmatrix} 0 & 0 & z_{cg} \end{bmatrix}$$

$$\mathbf{g}(\boldsymbol{\eta}_b) = \begin{bmatrix} [\mathbf{0}]_{3 \times 1} \\ mg \cos \theta \sin \phi \cdot z_{cg} \\ mg \sin \theta \cdot z_{cg} \\ -mg \cos \theta \sin \phi \cdot z_{cg} \end{bmatrix}$$



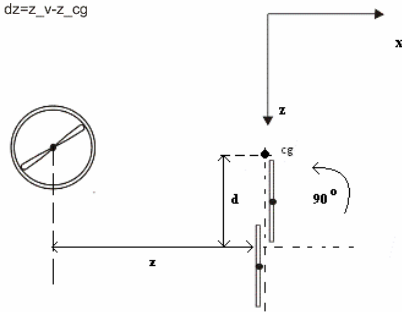
Kinematički model potisnika

- sile i momenti u koordinatnom sustavu balona (konfiguracija aktuatora)



$$\tau_b = [F_x \quad F_y \quad F_z \quad N_x \quad N_y \quad N_z]^T$$

Pozitivan smjer osi z!

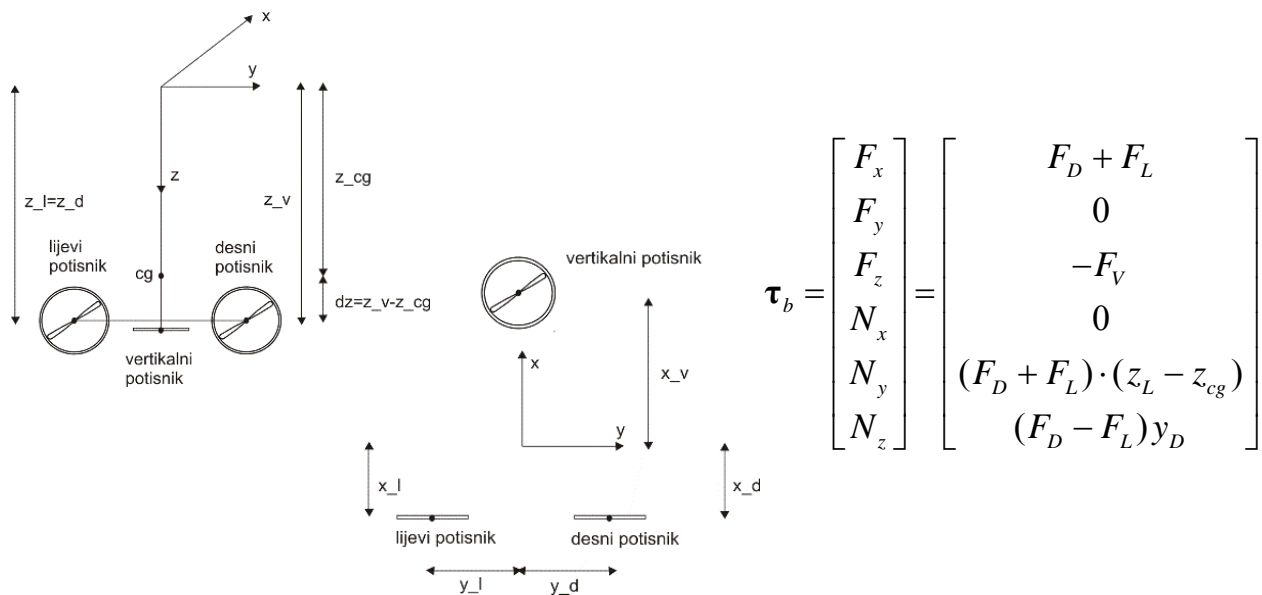


$$\tau_b = \begin{bmatrix} F_x \\ F_y \\ F_z \\ N_x \\ N_y \\ N_z \end{bmatrix} = \begin{bmatrix} (F_D + F_L) \cos \varphi \\ 0 \\ (F_D + F_L) \sin \varphi \\ 0 \\ (F_D + F_L) \cos \varphi \cdot d \\ F_r \cdot z \end{bmatrix}$$



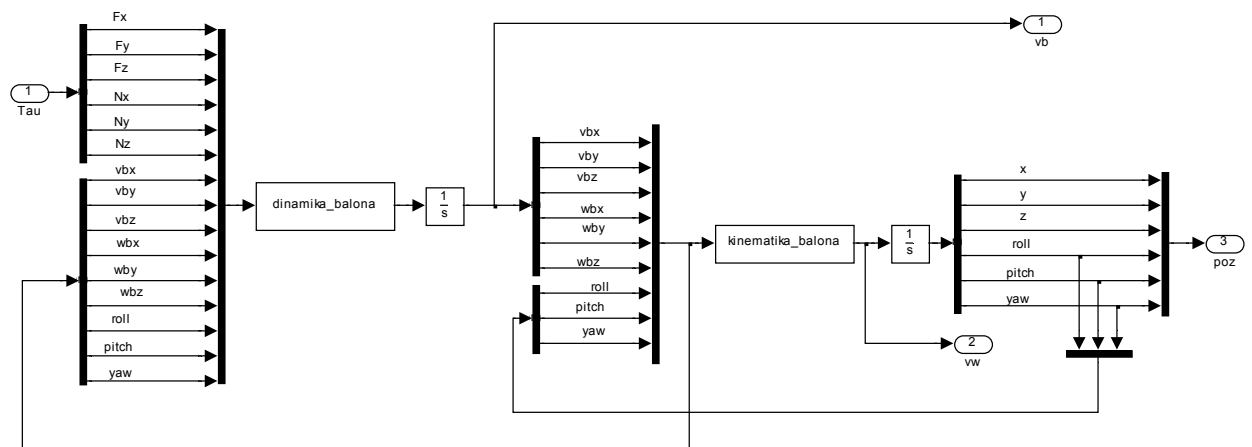
Kinematički model potisnika

- sile i momenti u koordinatnom sustavu balona (konfiguracija aktuatora)





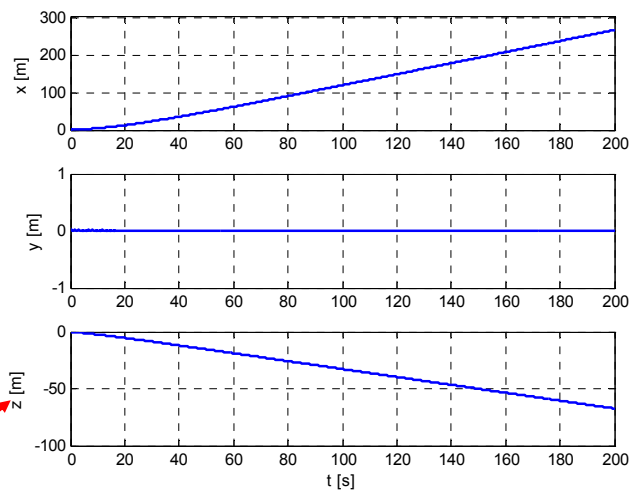
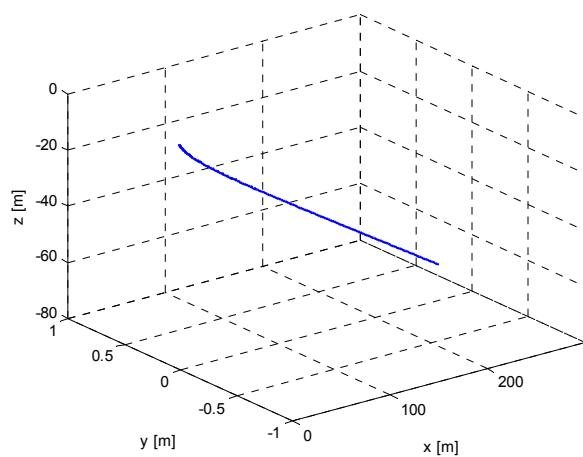
Model u Simulinku





Simulacija modela u Simulinku

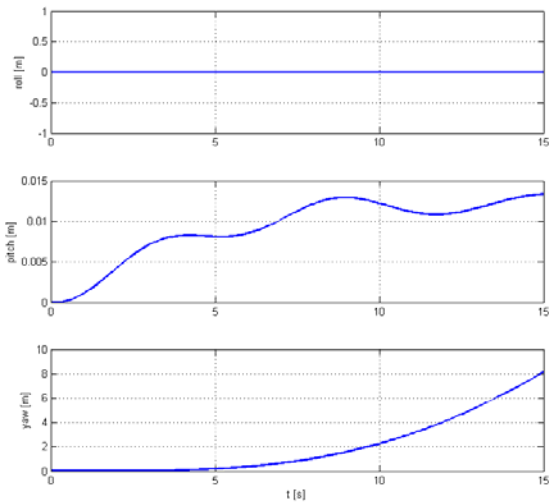
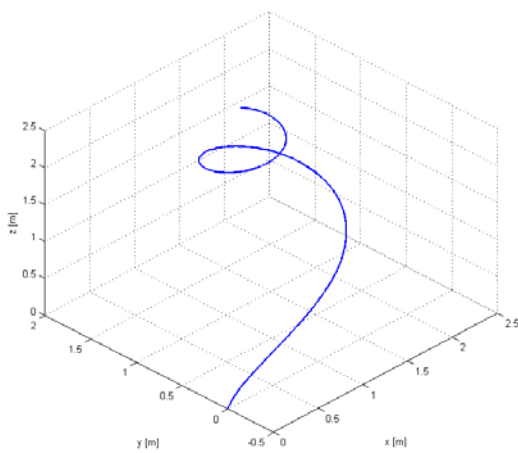
$$F_L = F_D = 0.0125 \text{ [N]} \quad - \text{ gibanje u smjeru osi } x$$





Simulacija modela u Simulinku

$F_L = F_D = 0.0125 [N]$, $F_r = 0.0125[N]$ - gibanje u smjeru osi x + skretanje





Matematički modeli letjelica - helikopter



Ikarus Piccolo

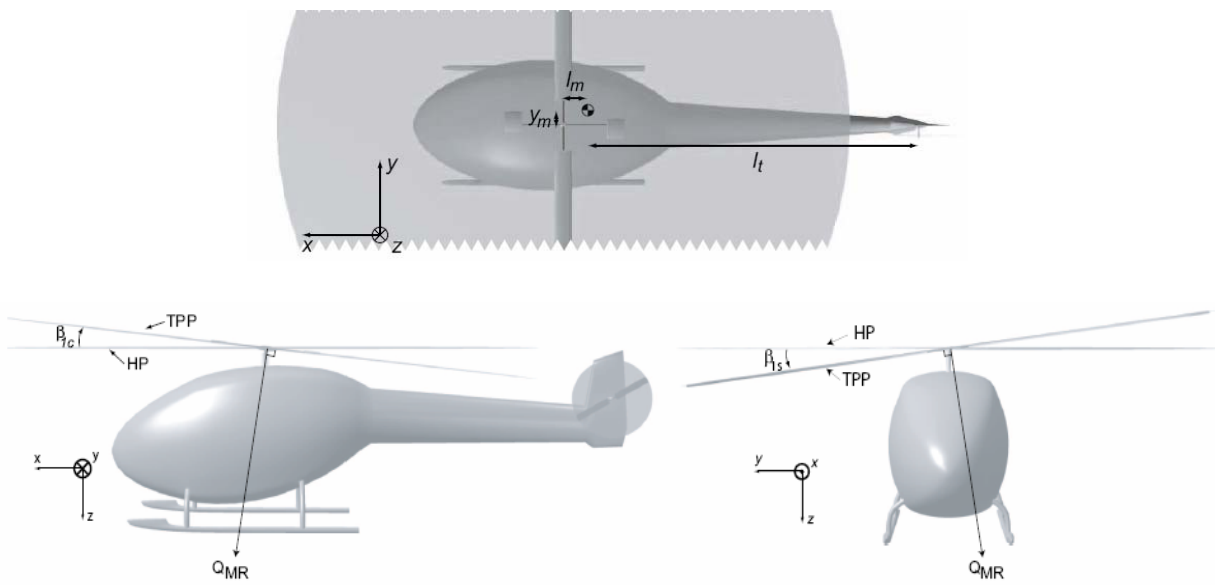


Yamaha R-50





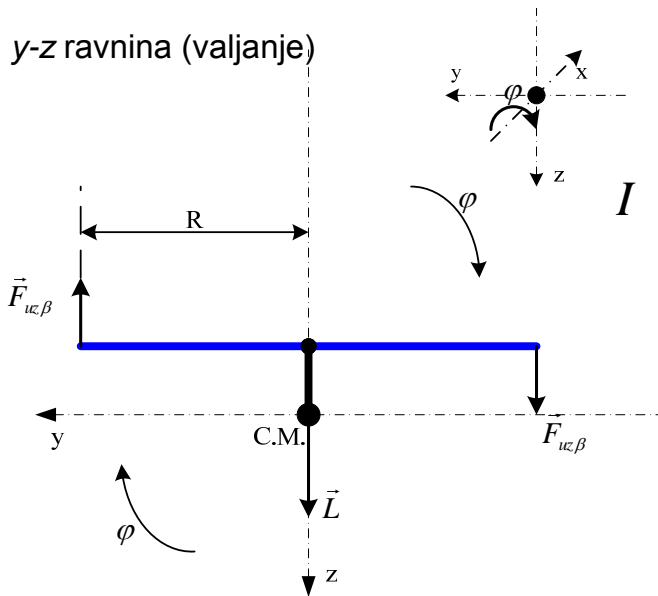
Matematički modeli letjelica - helikopter





Jednadžbe gibanja helikoptera

y-z ravnina (valjanje)



Moment pomoćnih lopatica

$$I \cdot \ddot{\varphi} = \tau_{uz\beta} - \tau_{\mu x} - \tau_{Dx}$$

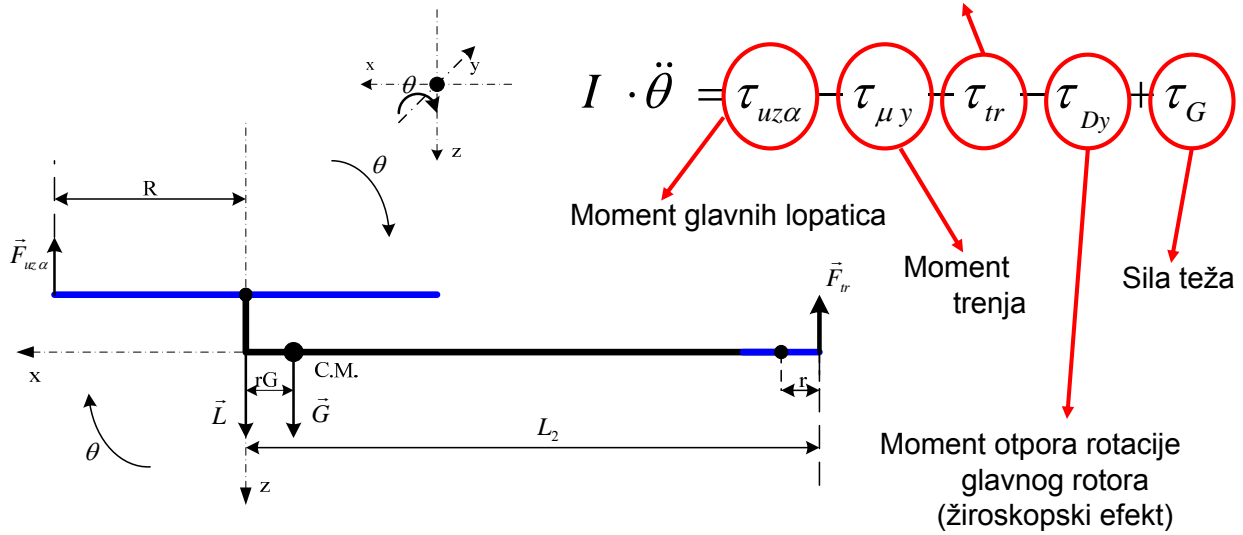
Moment trenja

Moment otpora rotacije
glavnog rotora
(žiroskopski efekt)



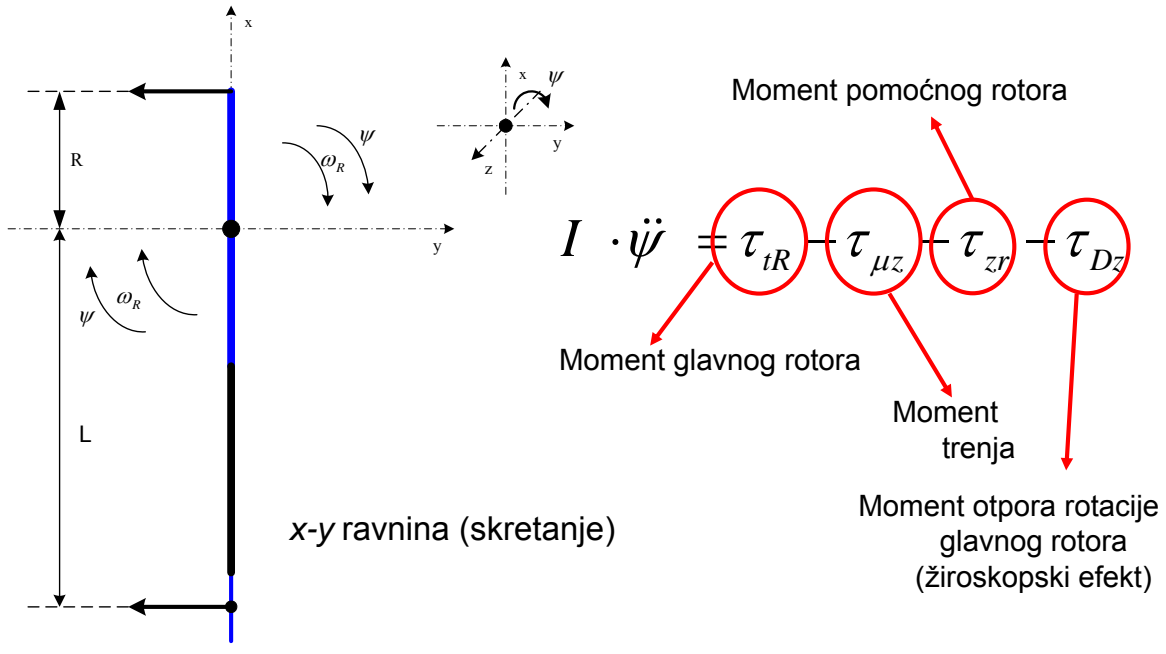
Jednadžbe gibanja helikoptera

x-z ravnina (poniranje)





Jednadžbe gibanja helikoptera





Jednadžbe gibanja helikoptera

$$\begin{bmatrix} F_{zx} \\ F_{zy} \\ F_{zz} \end{bmatrix} = \mathbf{R}_{z,\psi} \mathbf{R}_{y,\theta} \mathbf{R}_{x,\varphi} \mathbf{F} = \begin{bmatrix} \cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi \\ \cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi \\ \cos \varphi \cos \theta \end{bmatrix} \cdot F$$

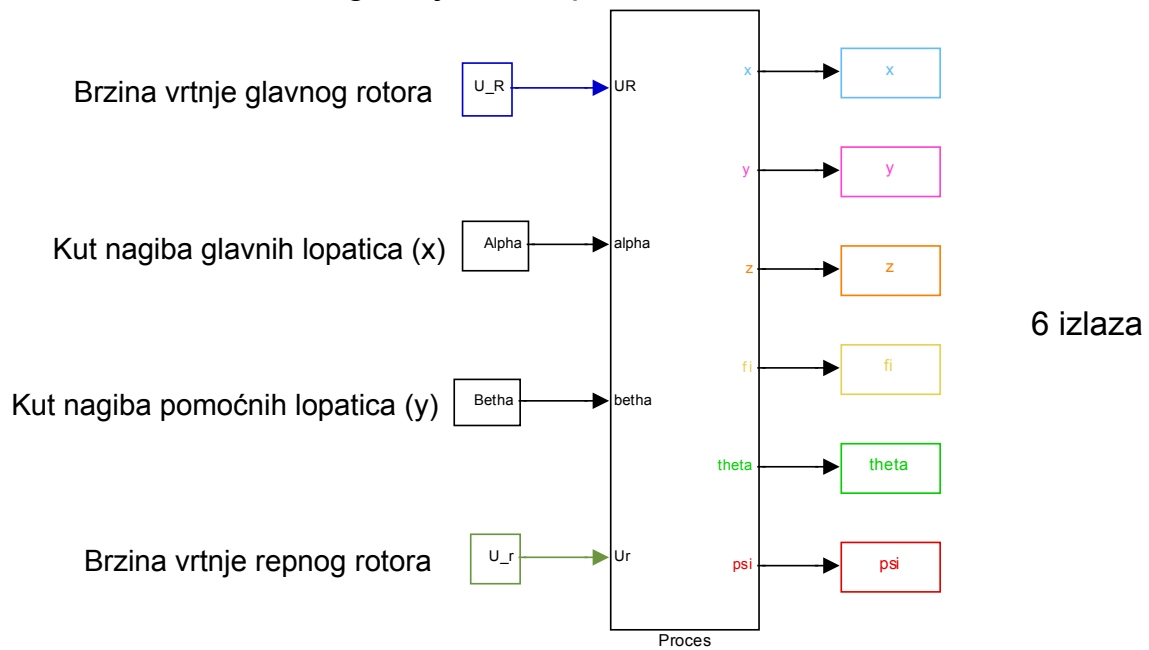
$$m\ddot{x} = F_{zx} - F_{\mu x}$$

$$m\ddot{y} = F_{zy} - F_{\mu y}$$

$$m\ddot{z} = F_{zz} - F_{\mu z} - G$$



Jednadžbe gibanja helikoptera





Sprege medju varijablama

- valjanje

$$I \cdot \ddot{\phi} = \tau_{uz\beta} - \tau_{\mu x} - \tau_{Dx} \Rightarrow G_{R\phi}, G_{\beta\phi}$$

- poniranje

$$I \cdot \ddot{\theta} = \tau_{uz\alpha} - \tau_{\mu y} - \tau_{tr} - \tau_{Dy} + \tau_G \Rightarrow G_{R\theta}, G_{r\theta}, G_{\alpha\theta}$$

- skretanje

$$I \cdot \ddot{\psi} = \tau_{tR} - \tau_{\mu z} - \tau_{zr} - \tau_{Dz} \Rightarrow G_{R\psi}, G_{r\psi}$$

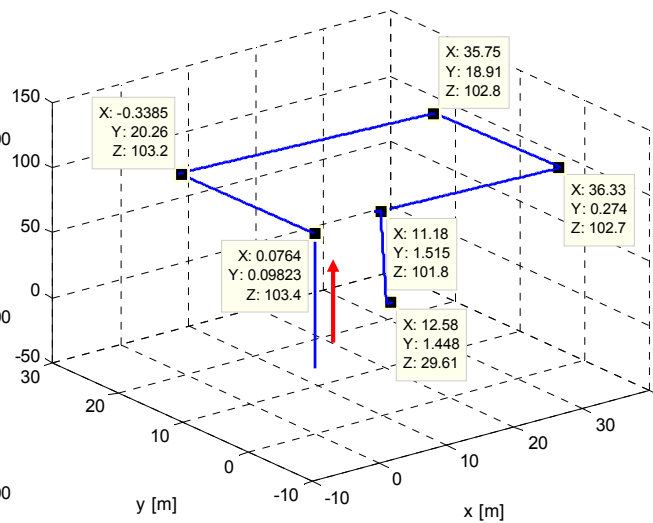
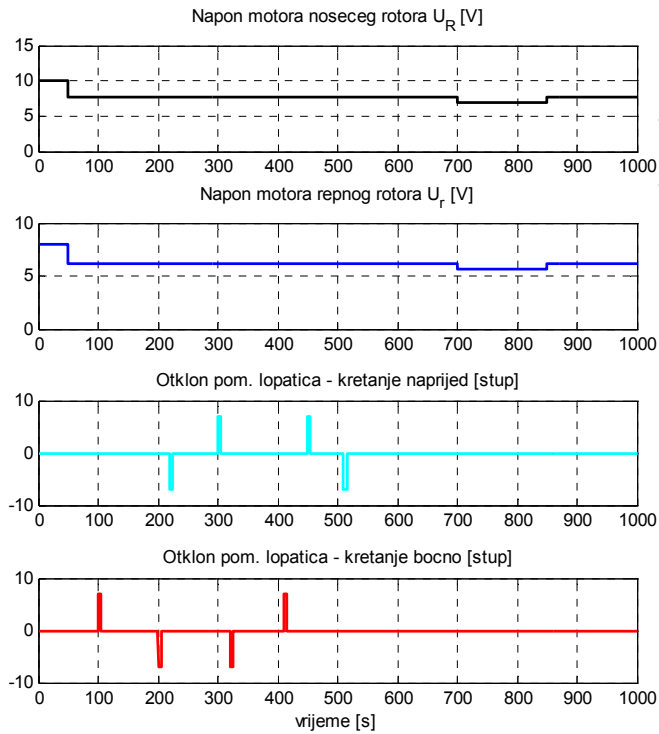
$$m\ddot{x} = F_{zx} - F_{\mu x} \Rightarrow G_{Rx}, G_{\beta x}$$

$$m\ddot{y} = F_{zy} - F_{\mu y} \Rightarrow G_{Ry}, G_{\alpha y}$$

$$m\ddot{z} = F_{zz} - F_{\mu z} - G \Rightarrow G_{Rz}$$



Simulacija modela u Simulinku

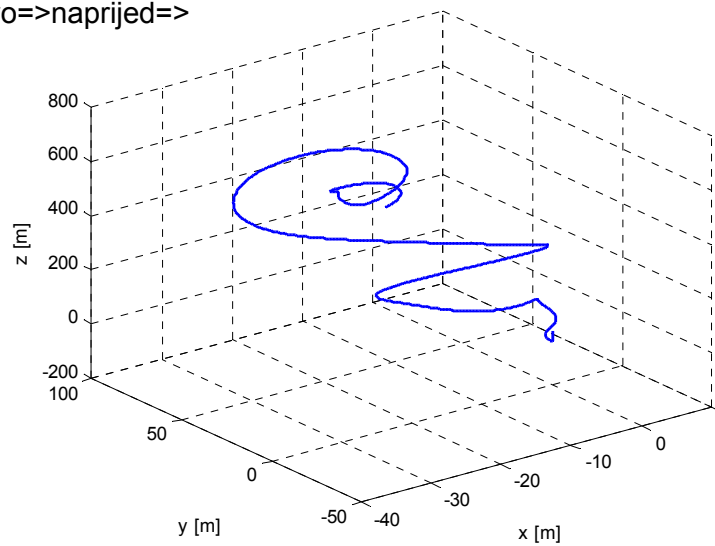


Niz naredbi
vert. gore=>unatrag=>lijevo=>naprijed=>
desno=>vert. dolje



Simulacija modela u Simulinku

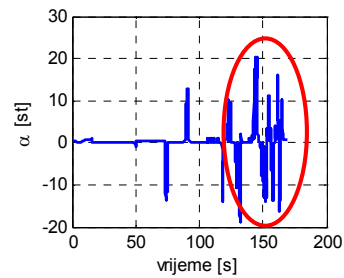
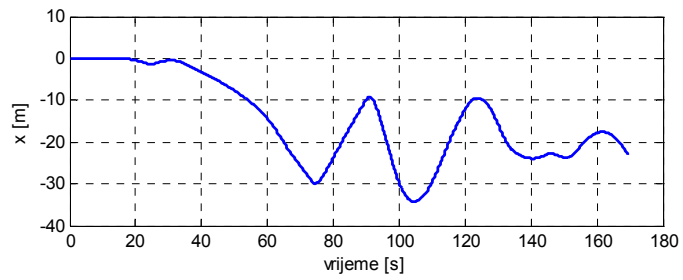
Operater putem upravljačke palice
vert. gore=>unatrag=>lijevo=>naprijed=>
desno=>vert. dolje





Simulacija modela u Simulinku

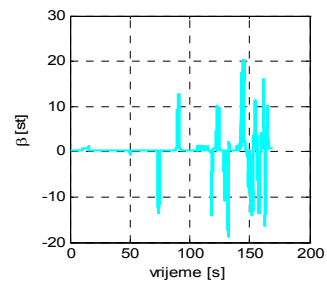
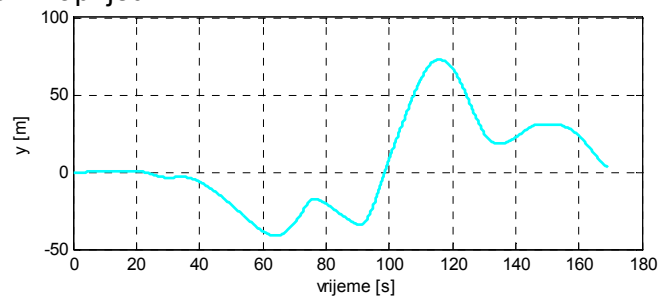
Operator putem upravljačke palice
vert. gore=>unatrag=>lijevo=>naprijed=>
desno=>vert. dolje





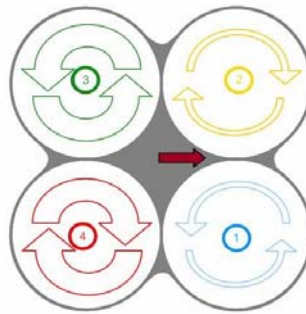
Simulacija modela u Simulinku

Operator putem upravljačke palice
vert. gore=>unatrag=>lijevo=>naprijed=>
desno=>vert. dolje



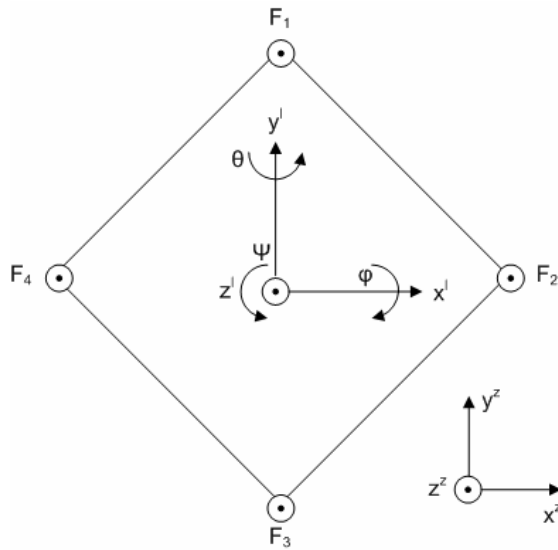


Matematički modeli letjelica - quadrotor





Matematički modeli letjelica - quadrotor



$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4$$



Jednadžbe gibanja quadrotora

- kutne brzine određene u koordinatnom sustavu letjelice

$$\vec{L}_\omega = \mathbf{I}\vec{\omega} \quad \text{količina kutnog gibanja}$$

$$\frac{\partial \vec{L}_\omega}{\partial t} = \frac{\partial L_\omega}{\partial t} \hat{\mathbf{L}}_\omega + \vec{\omega} \times \vec{L}_\omega = \frac{\partial \mathbf{I}}{\partial t} \vec{\omega} + \mathbf{I} \frac{\partial \vec{\omega}}{\partial t} + \vec{\omega} \times \mathbf{I}\vec{\omega}$$

Zbog simetričnosti sustava slijedi:

$$I_{xx} \frac{d\omega_x}{dt} = M_x + (I_{yy} - I_{zz}) \omega_y \omega_z$$

$$I_{yy} \frac{d\omega_y}{dt} = M_y + (I_{zz} - I_{xx}) \omega_x \omega_z$$

$$I_{zz} \frac{d\omega_z}{dt} = M_z + (I_{xx} - I_{yy}) \omega_x \omega_y$$

=0 – tijelo ne mijenja oblik
niti masu

nije uključeno trenje



Jednadžbe gibanja quadrotora

- veza između koordinatnih sustava

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \dot{\Psi} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\Psi) & \sin(\Psi) \\ 0 & -\sin(\Psi) & \cos(\Psi) \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\Theta} \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\Psi) & \sin(\Psi) \\ 0 & -\sin(\Psi) & \cos(\Psi) \end{bmatrix} \begin{bmatrix} \cos(\Theta) & 0 & -\sin(\Theta) \\ 0 & 1 & 0 \\ \sin(\Theta) & 0 & \cos(\Theta) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\Phi} \end{bmatrix}$$

- kutne brzine u nepomičnom koordinatnom sustavu

$$\begin{bmatrix} \dot{\Phi} \\ \dot{\Theta} \\ \dot{\Psi} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \quad \mathbf{J} = \begin{bmatrix} 1 & \sin(\psi)\tan(\theta) & \cos(\psi)\tan(\theta) \\ 0 & \cos(\psi) & -\sin(\psi) \\ 0 & \sin(\psi)/\cos(\theta) & \cos(\psi)/\cos(\theta) \end{bmatrix}$$



Jednadžbe gibanja quadrotora

- veza između sila i pozicije

$$\vec{L} = m_L \vec{v} \quad \text{količina gibanja}$$

$$\frac{\partial \vec{L}}{\partial t} = \frac{\partial m_L}{\partial t} \vec{v} + \frac{\partial \vec{v}}{\partial t} m_L = \frac{\partial m_L}{\partial t} \vec{v} + \frac{\partial v}{\partial t} \hat{v} \cdot m_L$$

=0 – tijelo ne mijenja masu

$$m_L \ddot{x} = |\mathbf{F}_R| (c\varphi s\theta c\psi + s\varphi s\psi) - F_{tx}$$

$$m_L \ddot{y} = |\mathbf{F}_R| (s\varphi s\theta c\psi - c\varphi c\psi) - F_{ty} \quad \text{uključeno trenje}$$

$$m_L \ddot{z} = |\mathbf{F}_R| (c\theta c\psi) - m_L g - F_{tz}$$

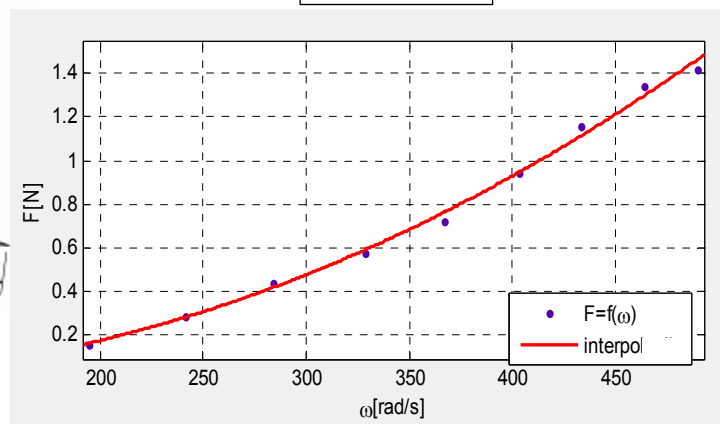
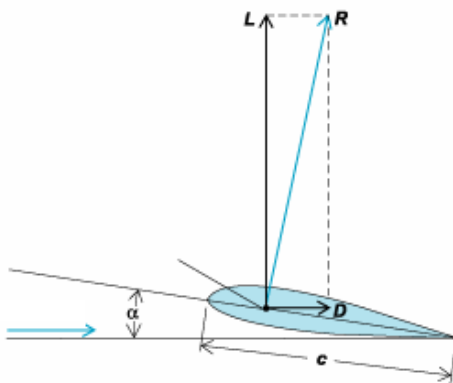


Matematički modeli letjelica – aktuator

propulzor (rotor/propeler)

Prvo približenje (potisna sila)

$$F \approx K \cdot \Omega^2$$





Analiza rada aktuatora

Pretpostavke:

- potisak je jednolično raspoređen po cijeloj površini koju zauzima rotor kada se vrti,
- rotacija strujanja zraka se zanemaruje,
- tok zraka je jasno odvojeni dio unutar zračnog prostora izvan kojeg je zrak savršeno miran.

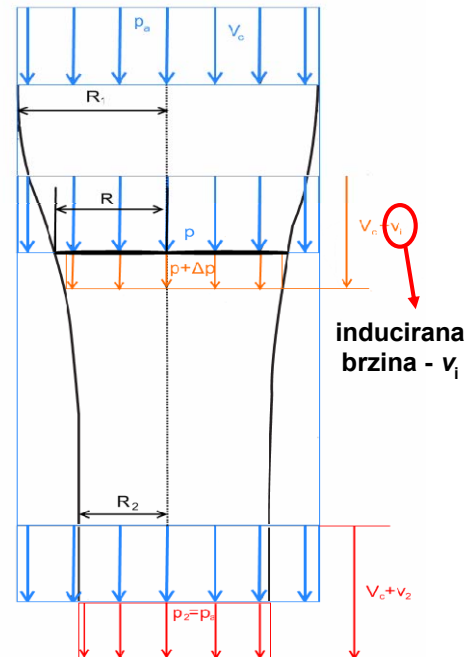
$$Q_u = \pi R_1^2 V_c$$

$$Q_i = \pi (R_1^2 - R_2^2) V_c + \pi R_2^2 (V_c + v_2)$$

$$Q_u - Q_i = \pi R_2^2 v_2 \neq 0 \quad ?$$

zakon o održanju mase!

Tok zraka kroz rotor letjelice pri vertikalnom uzlijetanju





Analiza rada aktuatora

$$Q_u - Q_i = \pi R_2^2 v_2 \neq 0$$

Do razlike dolazi zbog količine zraka koja u vremenskoj jedinici uđe u protok kroz njegovu bočnu stranu.

$$L_u = \rho V_c (\pi R_1^2 V_c + \pi R_2^2 v_2)$$

$$L_i = \rho \pi (R_1^2 - R_2^2) V_c^2 + \rho \pi R_2^2 (V_c + v_2)^2$$
 Promjena količine gibanja zraka.

Ta promjena mora biti izazvana nekom silom.

$$F = \Delta L = \rho \pi R_2^2 (V_c + v_2) v_2$$
 ← Potisak rotora.



Analiza rada aktuatora

$$F = \Delta L = \rho \pi R_2^2 (V_c + v_2) v_2$$

Tok zraka kontinuiran i nestlačiv \Rightarrow odnos v_2 i v_i , R_2 i R

$$\rho (V_c + v_i) R^2 \pi = \rho (V_c + v_2) R_2^2 \pi \Rightarrow (V_c + v_i) R^2 = (V_c + v_2) R_2^2$$

Korištenjem Bernoullijeve jednadžbe može se pokazati da vrijedi: $v_i = \frac{1}{2} v_2$

$$F = 2 \rho \pi R^2 (V_c + v_i) v_i$$

Napomena: medjuovisnost V_c i v_i !

Problem: mjerenje v_i nepraktično \Rightarrow mjeri se brzina vrtnje rotora

$$F = f(\Omega)$$