Rješenja 6.domaće zadaće iz Matematike 3E

1. $\vec{r}(t) = (4t+3)\vec{i} + 2\vec{j} + (14t-5)\vec{k},\ t \in [0,1]$

2. $\vec{r}(t) = (\cos t(1-\cos t))\vec{i} + \sin t(1-\cos t)\vec{j},\ t \in [0, 2\pi]$

3. a) $e^2\vec{i} + 1/2\vec{j}$
b) $\cos 1\vec{i} - \vec{k}$

4. a) $\sin t\vec{k}$
b) $e^t\vec{i} + \frac{1}{t}\vec{j} + \frac{1}{1+t^2}\vec{k}$
c) $e^t\vec{i} + e^t(t+1)\vec{j} + e^t(2t+t^2)\vec{k}$

5. a) $\vec{f}'(t) = 2e^{2t}\vec{i} - \cos t\vec{j},\ \vec{f}''(t) = 4e^{2t}\vec{i} + \sin t\vec{j}$
b) $\vec{f}'(t) = -2t\vec{i} + e^t(t+1)\vec{k},\ \vec{f}''(t) = -2\vec{i} + e^t(t+2)\vec{k}$
c) $\vec{f}'(t) = 2t\vec{j},\ \vec{f}''(t) = 2\vec{j}$

6. a) $I = (1-\cos 8)\vec{i} + \sin 8\vec{j} + 8\vec{k}$
b) $I = (2-\frac{5}{e})\vec{i} + \sqrt{2}(1-\frac{2}{e})\vec{j} + (1-\frac{1}{e})\vec{k}$

7. a) $\vec{g}(t) = (e^t(t-1)+4)\vec{i} + (\sqrt{t^2+1}+1)\vec{j} + (\frac{t^2}{2}+1)\vec{k}$
b) $\vec{g}(t) = (t+1)\vec{i} - \vec{j} + \frac{1}{3}t^3\vec{k}$
c) $\vec{g}(t) = e^{2t}(\vec{i} - \vec{k})$

8. a) $\frac{x-e}{e} = \frac{y-\frac{1}{e}}{-\frac{1}{e}} = \frac{z}{-1}$
b) $\frac{x-2}{1} = \frac{y-2}{2} = \frac{z-2}{3}$
c) $\vec{r}(u) = 6\vec{a} + \vec{b} - 4\vec{c} + u(3\vec{a} - 4\vec{c})$

9. a) $x = \frac{1}{5} - \cos t, y = \frac{4}{5} + \cos t, z = \frac{\sqrt{5}}{2}\sin t, t \in [0, \pi]$
b) $x = 1 + \cos t, y = 2 + 2\sin t, z = 2 + 2\sin t, t \in [0, \pi]$
c) $x = t, y = \frac{3}{8}t^2, z = t, t \in \mathbb{R}$

10. a) $\frac{\partial \vec{a}}{\partial x} = 2x\vec{i} + yz\vec{j},\ \frac{\partial \vec{a}}{\partial y} = xz\vec{j},\ \frac{\partial \vec{a}}{\partial z} = xy\vec{i} + 2z\vec{k}$
b) $\frac{\partial \vec{a}}{\partial x} = 4xy\vec{i} + 5y\vec{j} + y^2\vec{k},\ \frac{\partial \vec{a}}{\partial y} = 2x^2\vec{i} + 5x\vec{j} + 2xy\vec{k},\ \frac{\partial \vec{a}}{\partial z} = \vec{0}$

11. a) $\nabla f = -\frac{1}{2}\vec{i} + \frac{3}{2}\vec{j}$
b) $\nabla f = -\frac{\sqrt{2}}{2}(\vec{i} + 2\vec{j} + \vec{k})$

12. a) $\nabla(\ln r) = \frac{\vec{r}}{r^2}$

b) $\nabla(\sin r) = \cos r \frac{\vec{r}}{r}$
c) $\nabla(e^r) = \frac{e^r}{r}\vec{r}$

13. a) $\frac{\partial f}{\partial \vec{s}}|_T = \frac{2\sqrt{6}}{3}$
b) $\frac{\partial f}{\partial \vec{s}}|_T = -3\sqrt{2}$

14. a) $\frac{\partial \vec{f}}{\partial \vec{s}}|_T = \sqrt{2}(-2\vec{i} + \vec{j})$
b) $\frac{\partial \vec{f}}{\partial \vec{s}}|_T = \frac{e\sqrt{2}}{2}(\vec{j} - \vec{k})$

15. $(\vec{r}\nabla)\vec{v} = 2(yz\vec{i} + xz\vec{j} + xy\vec{k})$

16. a) $\triangle f = 2y^3z^4 + 6x^2yz^4 + 12x^2y^3z^2$
b) $\triangle f = -\cos r - \frac{2}{r}\sin r$

17. a) $\nabla\vec{v} = 6$, $\nabla \times \vec{v} = \vec{0}$
b) $\nabla\vec{v} = \frac{1}{r^2}$, $\nabla \times \vec{v} = \vec{0}$

18. a) $\triangle\vec{v} = 0$
b) $\triangle\vec{v} = -\frac{2}{r^4}\vec{r}$

19. a) $-3\vec{a} \times \vec{r_0}$
b) 0

20. a) $4e^{2\sin r}(r\cos^2 r - r\sin r - \cos r)\frac{\vec{r}}{r}$
b) $-\frac{1}{r^3}(\vec{a}\vec{r})$