

Rješenja 2. međuspita iz Matematike 3E

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1. (2 boda)

$$\int_0^1 dy \int_0^{y^2} \sin\left(\frac{y^3+1}{2}\right) dx = \dots = \frac{2}{3}(\cos \frac{1}{2} - \cos 1)$$

2. (2 boda)

$$P = \int_0^3 dy \int_y^{4y-y^2} dx = \dots = \frac{9}{2}$$

3. (2 boda)

$$\int_{-1}^1 dy \int_0^{\sqrt{1-y^2}} \sqrt{x^2 + y^2} dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \int_0^1 r^2 dr = \dots = \frac{\pi}{3}$$

4. (2 boda)

Za iskaz i dokaz Teorema srednje vrijednosti integralnog računa vidi str.12 u knjižici Višestruki integrali .

5. (4 boda)

a) (1B) $J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$

b) (1B) $J = -\frac{1}{4\sqrt{u^2-v^2}}$

c) (2B) $\iint_D xy dx dy = \int_4^9 du \int_1^4 \sqrt{\frac{u+v}{2}} \sqrt{\frac{u-v}{2}} \frac{1}{4\sqrt{u^2-v^2}} dv = \dots = \frac{15}{8}$

6. (2 boda)

$$V = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \int_0^{2\cos\varphi} r dr \int_0^r dz = \dots = \frac{32}{9}$$

7. (3 boda)

$$\begin{aligned} \iiint_V y dx dy dz &= (\text{poopćene cilindrične koordinate}) \\ &= \int_0^{2\pi} d\varphi \int_0^1 3\sqrt{3}r dr \int_{-4}^{5-9r^2} y dy = \dots = -\frac{27\sqrt{3}\pi}{2} \end{aligned}$$

8. (3 boda)

$$\iiint_V (x^2 + y^2) dx dy dz = \int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{4}} \sin\theta d\theta \int_0^5 r^4 \sin^2\theta dr = \dots = 625\pi \frac{8-5\sqrt{2}}{6}$$