

Rješenja završnog ispita iz Matematike 3E

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1. (2 boda) $\text{grad}f = -\frac{1}{x}\tilde{i} + \frac{1}{y}\tilde{j} + \frac{1}{z}\tilde{k}$, $T(-\frac{1}{2}, -1, \frac{1}{3})$

2. (2 boda) $\frac{\partial f}{\partial \vec{s}} = \frac{1}{\sqrt{13}}(3 - 4yz)$

3. (2 boda) $\frac{\partial \vec{v}}{\partial \vec{s}}|_T = \frac{2}{3}\vec{i}$

4. (2 boda) $-\frac{1}{r^3}\vec{r} \cdot \vec{a}$

5. (3 boda) $\int_{\mathcal{K}} z dx + x dy = \int_0^{2\pi} (3 - 2 \sin t)(-2 \sin t) dt + \int_0^{2\pi} 2 \cos t \cdot 2 \cos t dt = \dots = 8\pi$

6. (2 boda)

Def: Za vektorsko polje $f = f_1\vec{i} + f_2\vec{j} + f_3\vec{k}$ kažemo da je potencijalno ako postoji skalarno polje $p = p(x, y, z)$ takvo da vrijedi $f = \text{grad} p$.

$\text{rot}f = \vec{0} \Rightarrow$ polje \vec{f} je potencijalno

$$p(x, y, z) = \int_1^x f_1(x, y, z) dx + \int_0^y f_2(1, y, z) dy + \int_0^z f_3(1, 0, z) dz + C = -\frac{1}{x} + zy + C$$

7. (3 boda) $\iint_S dS = \iint_{\Omega_{xy}} \frac{dxdy}{\sqrt{1-x^2}} = \dots = 2$

8. (3 boda) $\iint_S \sqrt{z} dy dz = \iint_{S_1, x>0} \sqrt{z} dy dz + \iint_{S_2, x<0} \sqrt{z} dy dz = \dots = 0$

9. (3 boda)

f je neparna funkcija $\Rightarrow a_n = 0$, $L = \pi$

$$b_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx = \frac{2}{\pi} \int_0^\pi (x - \pi) \sin nx dx = \dots = -\frac{2}{n}$$

Fourierov red $S(x) = -\sum_{n \geq 1} \frac{2}{n} \sin(nx)$

$$S(\frac{\pi}{2}) = 2 \sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{2k+1} = -\frac{\pi}{1} \Rightarrow \sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{2k+1} = -\frac{\pi}{4}$$

10. (4 boda)

a) (1 bod) f original i n puta diferencijabilan: $\mathcal{L}(f'(t)) = sF(s) - f(0)$,

$$\mathcal{L}(f^{(n)}(t)) = s^n F(s) - s^{n-1} f(0) - \dots - f^{(n-1)}(0)$$

b) (3 boda) $i(t) = (\sin(3t) - \frac{1}{6} \cos(3t) + \frac{1}{6})S(t)$

11. (4 boda) $\iiint_V (x^2 + y^2) dx dy dz = \int_0^{2\pi} d\varphi \int_0^2 r^3 dr \int_{\frac{r}{2}}^2 dz = \dots = \frac{16}{3}\pi$

12. (4 boda)

$$\oint_K y dx + z dy + x dz = \iint_S (0-1) dy dz + (0-1) dx dz + (0-1) dx dy = \dots = -a^2 \pi \sqrt{2}$$