Rješenja ponovljenog završnog ispita iz Matematike 3E

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1. (**2 boda**) a) $\vec{f} = (\frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z})\vec{i} + (\frac{\partial f_1}{\partial z} - \frac{\partial f_3}{\partial x})\vec{j} + (\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y})\vec{k}$
b) $\mathrm{rot}\tilde{\mathrm{a}} = -\mathrm{xy}\tilde{\mathrm{i}} + \mathrm{yz}\tilde{\mathrm{k}}$

2. (**2 boda**) $T(\frac{5}{2}, 5, z),\ z \in \mathbb{R}$

3. (**2 boda**) $\triangle\vec{v} = (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2})(x^2\vec{i} + xy\vec{j} + xz^3\vec{k}) = 2\vec{i} + 6xz\vec{k},$
$\triangle\vec{v}|_T = 2\vec{i} - 6\vec{k}$

4. (**2 boda**) $8\vec{r}$

5. (**3 boda**) $I = 4\int_0^\pi (2\cos^2 t + \sin^2 t)dt = \ldots = 6\pi$

6. (**3 boda**) $I = \int_{\overline{AB}} + \int_{\overline{BC}} + \int_{\overline{CA}} = \int_0^1 (27t^2 - 108t + 45)dt = \ldots = 0$

7. (**3 boda**) $\iint_S x dS = \iint_{\Omega_{xz}} x\sqrt{4x^2+1}dxdz = \ldots = \frac{1}{24}(5\sqrt{5} - \frac{11}{5})$

8. (**3 boda**) $\iint_S x^2 dydz + ydxdz + (z^2+1)dxdy = \iint_{\Omega_{yz}}(1 - y^2 - z^2)dydz - \iint_{\Omega_{yz}}(1 - y^2 - z^2)dxdz + 2\iint_{\Omega_{xz}} \sqrt{1 - x^2 - z^2}dxdz + \iint_{\Omega_{xy}}(2 - x^2 - y^2)dxdy = \ldots = 0 + \frac{2\pi}{3} + \frac{3\pi}{2} = \frac{13\pi}{6}$

9. (**3 boda**)
f je parna funkcija $\Rightarrow b_n = 0,\ L = 1$
$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx = 2\int_0^1 \cos x\cos n\pi x\,dx = \ldots = \frac{2\sin 1\cos n\pi}{1-n^2\pi^2}$
Fourierov red $S(x) = 2\sin 1 + 2\sin 1\sum_{n\geq 1}\frac{\cos k\pi}{1-k^2\pi^2}\cos(kx)$

10. (**3 boda**)
a) (1 bod) $F(s) = \frac{48s}{(9s^2+1)^2}$ b) (2 boda) $f(t) = (1 - e^{-t+5})u(t-5)$

11. (**5 bodova**)
a) (1 bod) $J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$
b) (1 bod) $J = r$
c) (3 boda) $\iint_D (x^2+y^2)dxdy = \int_0^{2\pi} d\varphi\int_0^1 (r^2\cos^2\varphi + (r\sin\varphi + 1)^2)r\,dr = \ldots = \frac{3\pi}{2}$

12. (**4 boda**)
$\iint_S (x-y)dydz + zdxdy = \iiint_V 2dxdydz = 2ab\int_0^{2\pi} d\varphi\int_0^1 rdr\int_0^{r^2+2r\sin\varphi+1} dz = \ldots = 3ab\pi$