Robust Source Coding

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Overview

• Motivation for robust coding technologies
• Motivation / technology overview
• FEC and Multiple description coding
High Quality Now Less Natural

- Conventional circuit-switched networks
  - Virtually no bit errors, no loss

- Mobile networks
  - Reasonable cost implies bit errors

- Packet networks
  - Reasonable cost implies packet loss
Networks More Diverse

- How it was:
  - Single-paradigm network end-to-end
  - One service

- How it is:
  - Many paradigms in one composite network:
    - Circuit-switched network
    - Packet network
    - Wireless circuit-switched network
    - Wireless packet network
  - Many types of service
    - Range of quality-cost
    - Streaming versus one-on-one communication
Basic Technologies

- Bit-error correction
  - Established technology
  - Add redundancy
    - Parity bits
    - Reed-Solomon code

- Packet-loss recovery
  - New technology
  - Forward error correction (erasure codes)
  - Multiple-description coding
  - Packet-loss concealment
Packet-Loss Recovery

- Transmitter-based technologies
  - Automatic repeat request
  - Forward error correction (FEC)
  - Multiple description coding (MDC)
  - Layered coding
  - Interleaving

- Receiver-based technologies

Note: traditionally robustification sits in the physical and in the transport (fourth) layer of OSI model; we like it to sit in the application layer; cross-layer interaction?
Media-Specific FEC

- Redundancy added to the bit stream to counter bit errors and packet loss
- Redundancy added at “coding unit” level
- Examples
  - Low-rate coder added to the bit stream
  - Repeat of sections of key portions of coded data (H.263+)
- Low complexity
- Low latency
- Designer selects where to add redundancy
- Loss of quality when packet is lost
- Generally heuristic, but same idea as MDC
  - No problems with feedback in system
Media-Independent FEC

- Add redundancy to *bit stream*:
  - Reconstruct perfectly up to certain loss rate
  - Catastrophic failure beyond that error rate
    - Not flexible

- Examples
  - XOR
  - Reed Solomon

- Based on mathematics of finite fields
  - Rigorous
Multiple Description Coding (MDC)

- Transmit multiple descriptions
  - Optimize encoders to maximize expected performance with loss rate of channel

- Many decoders
  - Example: two-descriptions A and B
    - Decoder for A
    - Decoder for B
    - Decoder for A and B

- Operates on quantizer level / rooted in math
FEC versus MDC

- FEC operates on bit stream
  - Distortion measure irrelevant
  - Catastrophic failure
  - Convenient for legacy coders

- MDC operates on source
  - Minimizes distortion measure
  - Multiple decoding quality levels
  - Requires redesign of coder

- Conclusion: MDC should be better but requires redesign
FEC versus MDC

- **FEC (erasure code)**

  \[ x \xrightarrow{encoder} \xrightarrow{i} \text{add redundancy} \xrightarrow{network} i \xrightarrow{error correct} \xrightarrow{decoder} \xrightarrow{\hat{x}} \]

- **MDC**

  \[ x \xrightarrow{encoder} \xrightarrow{i} \text{MDC encoder} \xrightarrow{j_1, j_2} \text{network} \xrightarrow{j_1, j_2} \xrightarrow{\text{side decoder}} \xrightarrow{\hat{x}_1} \xrightarrow{\text{central decoder}} \xrightarrow{\hat{x}_c} \xrightarrow{\text{side decoder}} \xrightarrow{\hat{x}_2} \xrightarrow{\text{failure}} \xrightarrow{\hat{x}_f} \]
Simple MDC Scalar Quantizer

- Consider a CE quantizer with \( D = \sigma 2^{-r H(I)} \) = \( \sigma 2^{-2H(I)} \)
  - distortion decreases by \( 1/2 = 6dB \) per bit added

- Consider MDC that interleaves two CE quantizers
  - Receiving both descriptions decreases distortion by factor \( 1/2 = 6dB \) compared to single description
  - Small improvement, but FEC has none

\[
\begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 & 5 \\
\hline
0 & 1 & 2 & 3 & 4 & 5 \\
\hline
\end{array}
\]
Quantizer A
Quantizer B
Combination
Typical Two-Description MDC

- Joint codebook of two descriptions: \( \{c_{i_1}^{(1)}, c_{i_2}^{(2)}\}_{i_1 \in I_1, i_2 \in I_2} \)

- Central codebook: \( \{c_m^{(c)}\}_{m \in M} \)
  - (mappings must exist)

- Mappings: \( i_1 = i_1(m), \quad i_2 = i_2(m), \quad m = m(i_1, i_2) \)

- Average distortion is:
  - Probability both descriptions arrive: \( p_c \)
  - Probability description 1, 2, nothing arrives: \( p_1, p_2, p_0 \)
  
  \[ d = p_c d(x, c_m^{(c)}) + p_1 d(x, c_{i_1}^{(1)}) + p_2 d(x, c_{i_2}^{(2)}) + p_0 d_0 \]
Two-Description MDC: Operation

1. Get data value \( x \)

2. Find \( m \) that minimizes mean distortion:
\[
d = p_c d(x, c_m^{(c)}) + p_1 d(x, c_{i_1(m)}^{(1)}) + p_2 d(x, c_{i_2(m)}^{(2)}) + p_0 d_0
\]

3. Transmit
\[
i_1 = i_1(m), \quad i_2 = i_2(m)
\]

4. Decode:
   1. mapping \( m = m(i_1, i_2) \)
   2. if both arrive \( c_{m(i_1, i_2)}^{(c)} \)
   3. if 1 arrives \( c_{i_1}^{(1)} \)
   4. if 2 arrives \( c_{i_2}^{(1)} \)
Two Description Index Mappings

- **High packet loss solution**
  - Allows two similar descriptions

- **No packet loss solution**
  - Descriptions very dissimilar
Design Problem for Two Descriptions

• Constraints:
  – Constrained resolution
  – Constrained entropy
  – (structure joint codebook; lattice)
  – Mapping

• Minimize distortion (CR), fixed number of cells:
  \[ D = \mathbb{E}[\min_m p_c d(X, c_m^{(c)}) + p_1 d(X, c_{i_1(m)}^{(1)}) + p_2 d(X, c_{i_2(m)}^{(2)})] \]

• Minimize distortion under rate constraint (CE):
  \[ D = \mathbb{E}[p_c d(X, c_m^{(c)}) + p_1 d(X, c_{i_1(m)}^{(1)}) + p_2 d(X, c_{i_2(m)}^{(2)}) + \lambda_1 H(I_1(X)) + \lambda_2 H(I_2(X))] \]
How to Solve Design Problem

- Generalization of GLA (CR)
  1. Optimize encoder (one)
  2. Optimize decoders (three for two descriptions)
  3. If not converged go to 1

- Generalization of GLA (CE)
  1. Optimize encoder
  2. Optimize decoders
  3. Optimize code lengths
  4. If not converged go to 1
Example Multiple Description Coding

- Zhao/Kleijn 2004
Example Encoder Shifted Lattice MDC

\[
p=0.00\% \\
p=1.00\% \\
p=3.00\% \\
p=5.00\% \\
p=6.00\% \\
p=7.00\% \\
p=10.00\% \\
p=30.00\%
\]
Delays, Packets, and Redundancy

- ITU-T G.114: Voice: <100 ms is toll-quality; 150 ms reasonable limit
- Audio-visual: similar delay requirements
- Packet subject to Maximum Transfer Unit (MTU)
  - Typically 12 kbits
  - Blocks of 10-20 ms easily fit in MTU for audio and video
- Splitting blocks increases overhead
- Delay budget essentially gone with look-ahead, block size, processing, transmission and jitter buffer delays
- No delay available for redundancy; one block is reasonable
- Code two packets simultaneously, double the rate
Practical Comparison

- Two source packets, two redundant packets
  - Delay no more than two packets
- Gaussian source, R-D behavior: \( D = 2^{-2H(I)} \)

- Gilbert model to simulate packet network
Gilbert Model is Good

- 18000 calls between 9 sites

<table>
<thead>
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<th>(dB)</th>
<th>Gilbert</th>
<th>Direct</th>
<th>Gilbert</th>
<th>Direct</th>
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<tr>
<td>Bits/sample</td>
<td>1</td>
<td>1</td>
<td>9</td>
<td>9</td>
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<tr>
<td>FEC</td>
<td>5.81</td>
<td>5.77</td>
<td>26.96</td>
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<td>MDC</td>
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<td>optMDC</td>
<td>10.53</td>
<td>10.47</td>
<td>27.88</td>
<td>26.88</td>
</tr>
</tbody>
</table>
Practical FEC

- Distortion for FEC (Reed-Solomon):
  - At least 2 out of 4 arrive
  - 1 source packet arrives
  - None of the above

\[
D_{\text{FEC}} = p_{\text{FEC}} 2^{-2R} + p_{\text{os}} (1 + 2^{-2R}) + p_{\text{nd}} = v_1 2^{-2R} + v_2
\]

\[
v_1 = \frac{q}{p + q} (2 + 3p - 3p^2 - pq^2 + 3p^2q)
\]

\[
v_2 = \frac{q}{2(p + q)} (2 - 3q + 3pq + q^3 - 3pq^2)
\]
• Distortion for MDC
  - (Interleaved)
  - 2 out of 2 arrive
  - 1 out of 2 arrives
  - None arrive

\[ D_{\text{MDC}} = E[p_c d(X, c_m^{(c)}(X)) + p_1 d(X, c_i^{(1)}(X)) + p_0] \]

• Single channel versus Two channels (better)

\[
\begin{align*}
p_c &= \frac{q}{p+q} (1-2p + p^2 + pq) \\
p_1 &= \frac{2pq}{p+q} (2 - p - q) \\
p_0 &= \frac{p}{p+q} (1-2q + q^2 + pq)
\end{align*}
\]

\[
\begin{align*}
p_c &= \frac{q^2}{(p+q)^2} \\
p_1 &= \frac{2pq}{(p+q)^2} \\
p_0 &= \frac{p^2}{(p+q)^2}
\end{align*}
\]
Bound on Practical MDC II

- Distortion for MDC: \( D_{\text{MDC}} = E[p_c d(X, c_m^{(c)}(X)) + p_1 d(X, c_i^{(1)}(X)) + p_0] \)

- Rate-distortion bound (out of the blue here; Ozarow):

\[
D_C = 1 - \left( 1 - D_1 - \sqrt{D_1^2 - 2^{-4R}} \right) \geq 2^{-4R}, \quad \frac{2^{-4R}}{2} \leq D_1 \leq \frac{1}{2} (1 + 2^{-4R})
\]

- Where \( R \) is the source rate
• Informed versus non-informed case

Fig. 3. Achievable R-D bound and operating point of MDC ($R = 1$ b/sample).
Practical MDC vs FEC

- Single channel case
- Bounded by burst errors

Comparative rate-distortion performance of multiple description coding for real-time audiovisual communication over the Internet
Moo Young Kim; Kleijn, W.B.; Communications, IEEE Transactions on
Volume 54, Issue 4, April 2006 Page(s):625 - 636
Practical MDC vs FEC

- Two-channel case

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\[ \begin{align*}
\text{p} &= 0.005 \\
\text{q} &= 0.8 \\
\text{p} &= 0.1 \\
\text{q} &= 0.5
\end{align*} \]
Note on Current Implementations

- Proprietary coders common on Internet
  - Implementation not known
  - MDC likely used

- Usage of legacy coders
  - Addition of FEC layer convenient
Conclusions on MDC and FEC

- MDC minimizes distortion given packet-loss rate
  - MDC failure is inherently graceful
  - Full range of trade-offs
  - Requires new source coders

- FEC prevents info loss up to certain packet loss rate
  - Catastrophic failure
  - Can use legacy coders
  - Redesign difficult

- At high rates: quality constrained by burst errors
  - Consequence of delay constraint