# Zero-price Energy Offering by (Multiband) Robust Optimization

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### **Presentation outline**

Energy Offering under Market Price Uncertainty for a Price-Taker (EnOff-PT) A review of a highly cited Robust Optimization approach for the EnOff-PT A new Robust Optimization approach for the EnOff-PT Computational results Can we do better by Multiband Robust Optimization? All the presented results are strongly based on discussions with experts from our industrial partners, namely: A MAJOR EUROPEAN Uanle ELECTRIC UTILITY CONFIDENTIAL and are based on realistic data. The model and approach were validated by the Partners, as well.



# **The canonical Unit Commitment Problem (UC)**



# A different perspective: Energy Offering (EnOff)



### **Offering Curves**

- A company submits energy selling offers by specifying an offering curve for each of its generation unit and for each time period
- The offering curve is typically a (non-decreasing) step function



# **Energy Offering for a Price-Taker (EnOff-PT)**

producer that does not influence market price

(limited energy production)

The multi-unit offering problem can be decomposed into single-unit problems

For each unit of the producer :

**PRICE-TAKER** 

Given:

a planning horizon decomposed into a set T of time periods

the market price in each time period t

We want to:

choose the energy to offer in each time period in the market

So that:

- the total profit is maximized
- technical constraints of the units are satisfied (e.g., min up/down time, ramp limits)

### A natural formulation for the EnOff-PT





### **Price uncertainty in the EnOff-PT**





- protection entails the so-called Price of Robustness

### The Bertsimas-Sim $\Gamma$ -Robustness model (BS)

$$\max \sum_{j \in J} c_j x_j$$
$$\sum_{j \in J} a_{ij} x_j \le b_i \quad i \in I = \{1, \dots, m\}$$
$$x_j \ge 0 \qquad \qquad j \in J = \{1, \dots, n\}$$

#### Assumptions:

- 1) w.l.o.g. uncertainty just affects the coefficient matrix
- 2) the coefficients are independent random variables following an unknown symmetric distribution over a symmetric range

Deviation range: each coefficient  $a_{ij}$  assumes value in the symmetric range  $a_{ij} \in [\bar{a}_{ij} - d_{ij}^{\max}, \bar{a}_{ij} + d_{ij}^{\max}]$ 

Row-wise uncertainty: for each constraint *i*,  $\Gamma_i \in [0, n]$  specifies the max number of coefficients deviating from  $a_{ij}$ 



# $\Gamma\text{-}\textbf{Robustness}$ for the price-uncertain EnOff-PT

### Remarks about the EnOff-PT:

data uncertainty only affects the objective function (uncertain price coefficients)

#### **Γ-Robust Counterpart:**

- Given: 4 the nominal price in each period  $\lambda_t^{\text{NOM}}$ 
  - $\blacksquare$  the worst deviation of price w.r.t. the nominal price in each hour  $d_t$
  - 4 the **number**  $\Gamma > 0$  of price deviations for which protection is required

The robust counterpart is:

$$\max \sum_{t \in T} \begin{bmatrix} \lambda_t^{\text{NOM}} p_t - c_t(p_t) \end{bmatrix} - \Gamma z - \sum_{t \in T} q_t \\ z + q_t \ge d_t p_t \quad t \in T \\ z \ge 0 \\ q_t \ge 0 \\ p_t \in P_t \quad t \in T \end{bmatrix} \xrightarrow{\text{ADDITIONAL VARIABLES AND CONSTRAINTS}} FROM ROBUST DUALIZATION SET }$$

# The Baringo-Conejo approach (1)

![](_page_11_Figure_1.jpeg)

# The Baringo-Conejo approach (2)

- For each step function k, we obtain a robust optimal solution
- The **robust optimal solutions are merged** to build one **energy offering curve** for each time period
- For each time period:

![](_page_12_Figure_4.jpeg)

The offering curve built for each time period are submitted to the Energy Exchange

![](_page_13_Figure_0.jpeg)

The approach presents several issues that have NOT been pointed out until our work

#### ISSUE 1: definition of offering curves that break market rules

An offering curve is built considering a high number of intermediate prices between the maximum and minimum prices (100 prices in experimentals tests)

Violation of the limit on the number of steps of a curve imposed by market rules

ISSUE 2: risk of non-acceptance

The offering curves risk to be NOT accepted in the market (minimum price asked for selling)

#### **ISSUE 3: compromised optimality and feasibility**

The offering curves defined merging distinct optimal robust solutions obtained for different assumptions on the prices

optimality of energy production is compromised!

accepted portion of curves may result infeasible (e.g., violation of ramp constraints)

BIG

LOSSES

#### ISSUE 4: unnecessarily complex robust counterpart

The approach imposes full protection (worst price in each period)

it is not necessary to define the  $\Gamma$ -Robustness counterpart of increased dimension

# **Our revised approach based on** $\Gamma$ **-Robustness (1)**

### OUR OBJECTIVES:

- 4 (dramatically) increasing the chances that our energy offers are accepted
- defining robust solutions following the real spirit of Γ-Robustness (full protection is bad!)

### BASIC FEATURES OF OUR STRATEGY:

**4** we do not compete on price and **all our selling offers are at zero price** 

our offers are automatically accepted (  $\leq$  market price!)

- from historical market price data, we derive
  - the **nominal value** equals the **average price** over the past observations
  - the worst deviation is identified by excluding the worst M observations in a way that better fits the practice of power system professionals
- we exclude extreme unlikely price shortfalls and we show that partial protection grants (much) higher profits

# **Our revised approach based on \Gamma-Robustness (2)**

Given a set of past observations of the price for each time period:

- **4** the **nominal value** equals the **average price** over the past observations
- the worst deviation is identified by excluding the worst M observations

We do not want to be too conservative!

> Exclude protection against extreme and unlikely shortfalls

38 excluded

### EXAMPLE:

![](_page_15_Picture_7.jpeg)

- The worst deviation is defined excluding the worst 10% of observations ———
  - 44 is the worst relevant observation
  - - 4.9 is the worst deviation

Approach discussed and validated with industrial partners 🛕

### **Computational tests**

#### Tests on 45 realistic instances:

- 15 power plants located in 3 distinct Italian price-zone
- 24 time periods (= hours in one day)
- 3 percentages of exclusions of worst price observations (0, 10, 20 %)
- Experiments on a Windows machine with Intel 2 Duo-3.16 GHz processor and 8 GB of RAM
- Robust model coded in C/C++ interfaced through Concert Technology with CPLEX 12.5.1

Historical data and test period construction:

- For each hour:
  - we consider the prices observed in the price zone in a time window of 4 weeks
  - from these prices, we derive the nominal value and the max deviation of the uncertain price
- We compute the robust optimal solution for each Γ=0 (=no protection), 1, 2, ..., 24 (= full protection)
- We test the performance of the computed robust optimal solution in the week following the 4 weeks of the construction set
- The 4-week time window is shifted through the entire year with steps of 1 week providing 24 evaluation periods

### **Computational results**

|         |           | $\Gamma$ Best w.r.t. $\Gamma = 0$ |               | $\Gamma$ Best w.r.t. $\Gamma = 24$ |               |
|---------|-----------|-----------------------------------|---------------|------------------------------------|---------------|
| Unit ID | %Excluded | $\Delta \pi(EUR)$                 | $\Delta\pi\%$ | $\Delta \pi(EUR)$                  | $\Delta\pi\%$ |
| U1      | 0         | +40399                            | + 5.75        | + 213730                           | + 40.45       |
|         | 10        | +44350                            | + 6.32        | + 183161                           | + 32.54       |
|         | 20        | + 41921                           | + 5.97        | + 152608                           | +25.82        |
| U2      | 0         | + 23394                           | + 5.00        | + 333543                           | + 212.07      |
|         | 10        | + 46063                           | +9.85         | + 234371                           | + 83.96       |
|         | 20        | + 42071                           | + 9.00        | + 218607                           | +75.15        |
| U3      | 0         | - 1383                            | - 0.02        | + 1984511                          | +47.59        |
|         | 10        | +88980                            | + 1.44        | + 1031465                          | + 19.78       |
|         | 20        | + 105253                          | + 1.70        | + 627146                           | + 11.13       |
| U4      | 0         | + 43246                           | + 6.27        | + 255124                           | + 53.50       |
|         | 10        | + 57386                           | + 8.33        | + 181356                           | + 32.11       |
|         | 20        | + 51614                           | +7.49         | + 148634                           | +25.12        |
| U5      | 0         | + 15454                           | + 3.57        | + 340567                           | + 319.00      |
|         | 10        | + 45327                           | + 10.49       | + 240506                           | + 101.61      |
|         | 20        | +45331                            | + 10.49       | + 199406                           | +71.78        |
| U6      | 0         | + 14273                           | + 5.30        | + 2030185                          | +44.87        |
|         | 10        | +91766                            | + 10.58       | + 1117143                          | + 20.25       |
|         | 20        | + 152707                          | + 11.77       | + 675172                           | + 11.22       |
| U7      | 0         | + 307690                          | + 5.73        | + 1312795                          | + 30.13       |
|         | 10        | + 268508                          | + 5.00        | + 909989                           | + 19.28       |
|         | 20        | + 195207                          | + 3.64        | +792081                            | + 16.62       |

In almost all cases we can:

- greatly increase the profit w.r.t. a practice that we observed among professionals (average price)
- dramatically increase the profit w.r.t. full protection

Generation units of increasing capacity

![](_page_17_Figure_6.jpeg)

# Using the Bertsimas-Sim model (BS) in practice

- In real-world problems, historical data about the deviations of the uncertain coefficients are commonly available
- The data can be easily used to build histograms representing the distribution of the deviations

![](_page_18_Figure_3.jpeg)

![](_page_18_Figure_4.jpeg)

- The behaviour of the uncertainty internally to the deviation range is completely neglected (focus on the extreme deviations)
- According to our past experiences, practitioners would definitely prefer a more refined representation of the uncertainty

![](_page_19_Figure_0.jpeg)

- strongly data-driven uncertainty set
- first proposed by Bienstock for Portfolio Optimization (2007)
- Later extended to Network Design (Bienstock & D'Andreagiovanni 2009)
- a general theoretical study was missing!

![](_page_19_Picture_5.jpeg)

OUR AIM HAS BEEN TO FILL SUCH GAP

### **Formalizing Multiband Uncertainty**

Focus on the coefficients  $a_{ij}$  of each constraint i (row-wise uncertainty)

![](_page_20_Figure_2.jpeg)

- $\textbf{ 4 K deviation values } -\infty < d_{ij}^{K^-} < \cdots < \ d_{ij}^0 = 0 \ < \cdots < d_{ij}^{K^+} < +\infty \ \text{ for each coefficient } \ a_{ij}$
- igstarrow K deviation bands such that each band k corresponds with range  $\ (d^{k-1}_{ij}, d^k_{ij}]$
- igstarrow Lower and upper bounds  $\ 0 \leq l_k \leq u_k \leq n$  on the number of coefficients deviating in each band k
- $\clubsuit$  No upper bound on band k = 0, i.e.  $u_0 = n$
- igstarrow There exists a feasible assignment  $\ \sum_{k\in K} l_k \leq n$

### **General example of construction**

- + Focus on the coefficients  $a_{ij}$  of each constraint i (row uncertainty)
- ullet For each coefficient  $\,a_{ij}$  , we have a number of past observations  $\,\hat{a}_{ij}$
- Compute the percentage deviation of an observation from the nominal value  $\frac{\hat{a}_{ij}-\bar{a}_{ij}}{\bar{a}_{ij}}\cdot 100$
- Build the histogram representing the distribution of the percentage deviations for the considered constraint

### Example

#### **OBSERVED DISCRETE DISTRIBUTION** (ALL COEFFICIENTS IN THE CONSTRAINT)

#### POSSIBLE MULTI-BAND SET FOR THE CONSTRAINT (assuming 100 coefficients in the constraint)

![](_page_21_Figure_8.jpeg)

### The max-deviation auxiliary problem under MB

NON-LINEAR ROBUST COUNTERPART

MILP

![](_page_22_Figure_2.jpeg)

DEV01

max

$$\begin{split} \sum_{j \in J} \sum_{k \in K} d_{ij}^k \; x_j \; y_{ij}^k & \text{dentify} \\ l_k &\leq \sum_{j \in J} y_{ij}^k \leq u_k \\ \sum_{k \in K} y_{ij}^k &\leq 1 \\ y_{ij}^k &\in \{0,1\} \end{split} \begin{array}{ll} k \in K & \text{bounds on the no.} \\ k \in K & \text{dentify} \\ \text{bounds on the no.} \\ \text{of coefficients} \\ \text{falling in band k} \\ \text{fall in band k} \\ \text{fall in band k} \\ \text{fall in b$$

### **The Robust Counterpart under MB**

### **PROPOSITION 1 (Büsing & D'Andreagiovanni 12)**

The polytope associated with (DEV01) is integral.

Proof based on showing that the coefficient matrix of (DEV01) is totally unimodular

#### **THEOREM 1 (Büsing & D'Andreagiovanni 12)**

The Robust Counterpart of (MILP) under multi-band uncertainty is equivalent to:

$$\max \sum_{j \in J} c_j x_j$$

$$\sum_{j \in J} \bar{a}_{ij} x_j \left[ -\sum_{k \in K} l_k v_i^k + \sum_{k \in K} u_k w_i^k + \sum_{j \in J} z_i^j \right] \leq b_i$$

$$i \in I$$

$$\left[ -v_i^k + w_i^k + z_i^j \geq d_{ij}^k x_j \\ v_i^k, w_i^k \geq 0 \\ z_i^j \geq 0$$

$$x_j \geq 0$$

$$x_j \in \mathbb{Z}_+$$

$$(RLP)$$

$$i \in I$$

$$i \in I$$

$$i \in I$$

$$i \in I, j \in J \\ j \in J$$

$$j \in J_{\mathbb{Z}} \subseteq J$$

- Proof based on exploiting the integrality of (DEV01) and strong duality
- If the original problem is linear, then also the counterpart is linear

# **Separation of Multiband Robustness Cuts**

GOAL: finding a robust optimal solution for multi-band set D through a cutting-plane algorithm

Separation problem

Given a solution  $x \in \mathbb{R}^{n-\lambda}_+ \times \mathbb{Z}^{\lambda}_+$ , is this solution robust feasible for constraint *i* ?

 $x \in \mathbb{R}^{n-\lambda}_+ \times \mathbb{Z}^{\lambda}_+$  robust feasible for i  $\longleftrightarrow \sum_{j \in J} \bar{a}_{ij} x_j + DEV_i(x, D) \le b_i$ 

If this condition does not hold and  $y^*$  is an optimal solution to (DEV01) then

$$\sum_{j \in J} \bar{a}_{ij} x_j + \sum_{j \in J} \sum_{k \in K} d_{ij}^k x_j y_{ij}^{*k} \le b_i$$

is a valid inequality for the original formulation and cuts off x (robustness cut)

#### **THEOREM 2 (Büsing & D'Andreagiovanni 12)**

Separating a robustness cut corresponds with solving a min-cost flow problem

Proof based on showing the 1:1 correspondence between integral flows and assignments y of (DEV01)

![](_page_24_Picture_11.jpeg)

# **Multiband Robustness – further results**

- Dominance among multiband uncertainty sets
- Special results for 0-1 Linear Programs
- (Strong) valid inequalities for Mixed Integer Linear Programs
- Uncertainty in right-hand-sides

4

Probability bounds of constraint violation

### **Multiband Robustness for Energy Offering**

MULTIBAND ROBUST COUNTERPART

$$\max \sum_{t \in T} [\lambda_t^{\max} p_t - c_t(p_t)] + \sum_{k \in K} \ell_k v_k - \sum_{k \in K} u_k w_k - \sum_{t \in T} q_t$$

$$-v_k + w_k + q_t \ge d_t^k p_t \qquad t \in T, k \in K$$

$$v_k \ge 0 \qquad k \in K$$

$$w_k \ge 0 \qquad k \in K$$

$$q_t \ge 0 \qquad t \in T$$

$$p_t \in P_t \qquad t \in T, k \in K$$

Preliminary computational results for a system of 5 deviation bands

Increase in profit of about 23% on average

### **Final Remarks**

We have addressed the Energy Offering Problem for a price-taker considering price uncertainty

We pointed out the limits of a highly-cited approach for solving the problem:

- risk of refusal of energy offers
- infeasibility and sub-optimality of energy offers
- excessive conservatism (full protection)
- We proposed an alternative approach that:
  - dramatically reduces the risk of non-acceptance of offers
  - better fits the spirit of Robust Optimization
  - grants in practice a (very) good increase in profit w.r.t. industry practice

#### FOR FURTHER DETAILS

- F. D'Andreagiovanni \*, G. Felici, F. Lacalandra,
- \* First Author

"Revisiting the use of Robust Optimization for optimal energy offering under price uncertainty" Submitted for publication, available on ArXiV

#### ONGOING WORK

Extension to realistic Price-Maker cases

![](_page_27_Picture_16.jpeg)