Concise Papers

Spread Spectrum Performance Analysis in Arbitrary Interference

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Abstract—An important application of spread spectrum modulation is to provide interference immune communications. Generally, the approach to system performance analysis will assume a host of various interference situations with the hope the results give wide applicability, while, based on the concept and properties of pseudo-random spreading waveforms, it is possible to rid the analysis of artificial or restricting interference assumptions and obtain completely general performance bounds.

I. INTRODUCTION

The basic principles of spread spectrum modulation and communications system properties rest on the pioneering work of C. Shannon (1948) and V. Kotelnikov (1947). Shannon showed that a communications system performance is directly related to the time-bandwidth-power product of the transmitted signal. Spread spectrum modulation is characterized by a large time-bandwidth product to achieve desirable performance of which interference immune communications will be our subject. Kotelnikov showed that in a white noise interference environment the detection or demodulation performance of a transmitted symbol only depends on the received symbol energy. Thus, for a system employing a large time-bandwidth product, there is great freedom in choice of the symbol waveform without affecting the performance.

Generally, communications system analysis presupposes a defined interference environment and often in statistical terms. However, this approach is not pleasing for most applications since spread spectrum modulation is used when the interference environment lacks a known characterization. The purpose of this paper is to eliminate artificial interference assumptions in pursuit of rigorous spread spectrum performance analysis.

II. COMMUNICATIONS MODEL

Spread spectrum modulation may be implemented in many ways but always with the transmission bandwidth significantly exceeding that of the information signal. This band spreading is most commonly achieved by multiplying (mixing) the information signal \( m(t) \) with a spreading waveform \( s(t) \) and transmit an amplified version of the signal

\[
\text{Re} \{ m(t)s(t) \exp (i\omega_0 t) \}
\]

centered about the carrier frequency \( f_0 = \omega_0/2\pi \). Now, to present the basic spread spectrum concepts most clearly, the performance investigation will be limited to binary antipodal signaling; that is, \( m(t) = \pm 1 \) for \( t \in (0, T) \) and \( m(t) = 0 \) otherwise, with \( T \) being the transmitted symbol duration. The spreading waveform may be either a continuous or discrete (hopping) form of phase, frequency, or time modulation. In any case, spreading waveforms having both in phase and quadrature phase components will be of direct interest, and an analysis of such waveforms is facilitated by a complex signal formulation. Furthermore, from an analysis point of view, it is convenient to normalize the spreading waveform to have unit symbol energy; that is, \( \int |s(t)|^2 \, dt = 1 \) with integration over the symbol period, \( (0, T) \). In the common case the spreading waveform has constant power; this normalization implies \( |s(t)| = \sqrt{1/T} \).

Assuming that the transmitted signal is sent over a linear, nondispersive medium, the received desired signal will be

\[
\sqrt{2E_b} \text{Re} \{ m(t)s(t) \exp (i\omega_0 t) \}
\]

where \( E_b \) equals the received bit energy over the symbol period \((0, T)\). [Note, \( |m(t)| = 1 \) so \( m(t)s(t) \) has unit energy over \((0, T)\).] In addition to the received desired signal, two interference signals will be considered; namely,

\[
W(t) = \sqrt{2N_0} \text{Re} \{ w(t) \exp (i\omega_0 t) \}
\]

\[
U(t) = \sqrt{2E_u} \text{Re} \{ u(t) \exp (i\omega_0 t) \}
\]

where \( w(t) \) represents a complex, white noise (Gaussian) process of unit (double-sided) spectral density. With this definition \( W(t) \) becomes a white noise process of spectral density \( N_0 \) (single-sided). The signal \( u(t) \) represents a square integrable (Lebesgue) signal of unit energy over the symbol period \((0, T)\). In this way \( U(t) \) will have the energy \( E_u \) over the symbol period.

As the first signal processing step, the composite received signal

\[
R(t) = \sqrt{2E_b} \text{Re} \{ m(t)s(t) \exp (i\omega_0 t) \} + W(t) + U(t)
\]

is multiplied (mixed) with the coherent reference

\[
\sqrt{2} \exp (-i\omega_0 t) = \sqrt{2} \cos (\omega_0 t) - i\sqrt{2} \sin (\omega_0 t)
\]

at the receiver followed by low pass filtering to remove the double carrier frequency components, thus obtaining the complex received signal

\[
r(t) = \{ R(t) \sqrt{2} \exp (-i\omega_0 t) \}_L = \sqrt{2E_b}m(t)s(t) + \sqrt{2N_0}w(t) + \sqrt{2E_u}u(t)
\]

[This result follows most conveniently from using the relation \( \text{Re}(z) = (z + z^*)/2 \) where \( * \) denotes complex conjugation.] The complex nature of the receive signal \( r(t) \) results from defining the signal of (4) in complex terms. The transmitted signal, as well as the received signal \( R(t) \), are real signals, although their mathematical formulation incorporates complex functions just for our convenience. The real part of \( r(t) \) will be referred to as the in phase and the imaginary part
as the quadrature component. In Figure 1 the spread spectrum communications system model is shown as a block diagram including the mathematical descriptions of the signal at various points.

This first step of receiver processing—to obtain the complex signal \( r(t) \) from the real received signal \( R(t) \)—was made without reference to some optionality criterion for the receiver signal processing. It is justified because this processing step is reversible [1]. Clearly, \( R(t) \) equals the real part of \( r(t) \exp(i\omega_0 t) \).

It is important to note that the interference signal \( \sqrt{E_u} u(t) \) is not necessarily assumed to have constant power. The only restriction is that \( \sqrt{E_u} u(t) \) is a square integrable signal with finite energy \( E \), over the symbol period. We have also separated the white noise signal from the interference signal \( \sqrt{E_u} u(t) \) to allow a rigorous analysis.

III. SYSTEM PERFORMANCE OBJECTIVE

A fundamental question for spread spectrum performance analysis is what performance criterion should be used. Although the detection signal-to-noise ratio for analog communication and the bit error probability for digital communication are natural performance measures, they do not by themselves completely define the criterion since it is not necessarily explicit what are the properties of the interference signal \( u(t) \). Generally, spread spectrum modulation is used in an unknown interference environment that either lacks a statistical or deterministic characterization or makes such an assumption artificial and most disconcerting from a system design point of view. Any particular assumption will always leave the uneasy feeling that the most critical case may not have been considered. Therefore, since often the subject of spread spectrum performance analysis is to ensure a minimum system performance under all conditions, it seems natural to define the best receiver structure as:

\[
\text{The optimum receiver processing that will maximize the minimum performance with the minimum taken over all finite energy interference signals.}
\]

Above \( u(t) \) was defined as a unit energy waveform, and thus \( \sqrt{E_u} u(t) \) has energy \( E_u \), without any further restrictions; therefore, the purpose will be to maximize the minimum of the particular performance measure over all unit energy waveforms \( u(t) \in L^2(0, T) \); i.e., Lebesgue square integrable functions over \( (0, T) \).

Since no statistical properties of the interference signal \( u(t) \) will be assumed it is not possible to determine the best receiver structure using the maximum likelihood approach. Therefore, lacking a general approach, we will limit our investigation to the best linear receiver processing structure. According to Riesz's theorem [2], the correlation integral represents the most general linear processing structure, so we will consider the functional

\[
z = \int_a^b r(t) g^*(t) \, dt = \mu + n
\]

where

\[
\mu = \sqrt{E_b} \int_a^b m(t) s(t) g^*(t) \, dt,
\]

\[
n = \sqrt{N_0} \int_a^b w(t) g^*(t) \, dt + \sqrt{E_u} \int_a^b u(t) g^*(t) \, dt,
\]

over the time interval \((a, b) \supset (0, T)\) where the white noise integral is not proper but well defined as a stochastic or Ito integral. Without loss of generality, we may assume \( g(t) \) to have unit energy over \((a, b)\).

First, if we consider \( u(t) \) to be a zero mean, random process of average unit energy over \((a, b)\) then the signal-to-noise ratio \( |\mu|^2 / E |n|^2 \) can be determined since

\[
E |n|^2 = N_0 + E_u \int_a^b g(t') K_u(t, t') g^*(t) \, dt \, dt'
\]

\[
\leq N_0 + E_u \lambda_u(a, b)
\]
where $\lambda_u(a, b)$ represents the maximum characteristic value of the covariance kernel $K_u(t, t') = E[u(t)u^*(t')]$ over $t, t' \in (a, b)$. [Supremum of the double integral over all unit energy functions defines $\lambda_u(a, b)$.] Since $\lambda_u(a, b) \geq \lambda_u(0, T)$, it follows that we may restrict the correlation interval $(a, b)$ to $(0, T)$. Then, according to Schwartz' theorem, $\| \mu \|^2$ is maximized for $g(t) = s(t)$ and one may be led to believe that the matched filter receiver is the best max-min receiver for a random interference signal $u(t)$. This reasoning is false. In fact, it is generally possible to select a $g(t)$ in (7) different from $s(t)$ for which the decrease in $E \| n \|^2$ outweighs the decrease in $\| \mu \|^2$ [3]. On the other hand, if the receiver correlation waveform $g(t)$ is given, $E \| n \|^2$ is well defined and $\| \mu \|^2$ is maximized by $s(t) = g(t)$.

So rather, a transmission matched to the receive correlation waveform is optimum. Even if this amounts to matched signal processing, conceptually there is a decisive difference because it implies that spread spectrum communications systems should be analyzed by first postulating the receiver processing structure rather than beginning with the transmitted signal and subsequently determining the optimum transmission.

**IV. SIGNAL-TO-NOISE PERFORMANCE**

With the best linear processing obtained for $s(t) = g(t)$, we are going to assume that initially $s(t) = g(t)$ was chosen as the receiver correlation signal in (6a). The present task will be to extend the signal-to-noise ratio performance to include a finite energy but otherwise completely arbitrary interference.

The performance analysis will be possible by considering the spreading waveform random. Since the spreading waveform can be designed with known statistical properties, the performance analysis will be well defined. However, this viewpoint leads to a conceptual problem: how is it possible to generate two identical random waveforms, one at the transmitter and one at the receiver, for the correlation processing. Now if they were truly random, it would be impossible, but one may use deterministic pseudo-random number or sequence generators to produce essentially random waveforms to justify this approach.

In statistical terms the spreading waveform will be defined to have zero mean and continuous covariance $K_s(t, t') = E[s(t)s^*(t')]$ and the existence of the "covariance" function $k_s(t, t') = E[s(t)s(t')]$; no complex conjugate of $s(t)$. With $g(t) = s(t)$ and $(a, b) = (0, T)$ in (6b) we obtain $\mu = \sqrt{E_n}$ real-valued and therefore consider the corresponding signal-to-noise ratio $\mu^2/Var \{ Re n \}$. The variance of $Re n$ is most conveniently obtained from the identity $Var \{ Re n \} = \frac{1}{2} E \| n \|^2 + \frac{1}{2} Re E_n^2$ valid for a zero mean, complex, random variable.

Explicitly, the expression for

$$ E \| n \|^2 = N_0 + \frac{E_u}{2} \int_0^T u^*(t)K_u(t, t')u(t') dt \ dt' $$

and thus alternatively to (7)

$$ E \| n \|^2 = N_0 + \frac{E_u}{2} \int_0^T S(f) df $$

being the Fourier transform of $u(t)$ over the symbol period $(0, T)$. The bound of (10) is obtained using Plancherel's identity [4] which implies that the integral $\int | U(f) |^2 df = 1$ over $(-\infty, \infty)$, since $u(t)$ is assumed to have unit energy over $(0, T)$. Clearly, the bound of (10) is only useful if the spectral density $S(f)$ exists for all frequencies; no $\delta$-functions.

In general, the "variance"

$$ En^2 = E_u \int_0^T u(t)K_u^*(t, t')u(t') dt \ dt' $$

lacks any alternative (spectral) representation. [There is no white noise since for a complex white noise process $E[u(t)u^*(t')] = 0$ for all $t, t'$.] Yet, we can make an important observation. Whatever the "covariance" function $k_s(t, t')$ may be, the contribution of $Re En^2$ to $Var \{ Re n \}$ may either be positive or negative depending on the interference signal $u(t)$. Therefore, it is advantageous to design a system with a spreading waveform for which $k_s(t, t')$ identically vanishes. Since we must have $K_s(t, t) = E \| s(t) \|^2 = 0$ for some $t \in (0, T)$, it is necessary to consider $s(t)$ complex. We will term such a waveform proper if $k_s(t, t') = 0$ for all $t, t' \in (0, T)$. Such spreading waveforms do exist in abundance, since $s(t) = x(t) + iy(t)$ will be proper if $x(t)$ and $y(t)$ are uncorrelated, zero mean, real signal process with the same statistical description. So for a proper complex spreading waveform $Var \{ Re n \} = \frac{1}{2} E \| n \|^2 = \frac{1}{2} N_0 + E_u \lambda_s(0, T)$. If, however, $s(t)$ is real $k_s(t, t') = k_s(t', t)$ and since $| En^2 | \leq E \| n \|^2 - N_0$ we may express $Var \{ Re n \} \leq \frac{1}{2} N_0 + E_u \lambda_s(0, T)$ where $0 \leq \alpha \leq 2$.

These results can be combined to stabilize the assured detection signal-to-noise ratio

$$ SNR = \frac{1}{2} \min \{ \mu^2/Var \{ Re n \} \} = E_b/[N_0 + E_u \alpha \lambda_s(0, T)] $$

where $\alpha = 1$ for a proper complex and $\alpha = 2$ for a real spreading waveform. When the spectral density exists for the spreading waveform $\lambda_s(0, T) = \max S(f)$. For example, if we consider discrete phase modulation (phase hopping) at a rate $\Delta^{-1}$ we have the familiar spectral density $S(f) = (\Delta/T) \sin((\pi f \Delta)/(\pi f \Delta))^2$ so max $S(f) = \Delta/T$. Note, we have normalized $s(t)$ to unit energy over $(0, T)$ and thus to power $1/T$; i.e., $s(t) = 1/T$.

In obtaining SNR of (13) we considered the maximum of
\(\alpha\lambda(0, T)\). The minimum of \([\alpha\lambda(0, T)]^{-1}\) is referred to as the spread spectrum processing gain. For discrete phase shift modulation it equals \(T/\Delta\) for a proper (M-ary PSK with \(M \geq 3\)) and \(T/2\Delta\) for a real (binary PSK) spreading waveform. This 3 dB advantage makes sense from a physical point of view since a proper spreading waveform will distribute the interference energy \(E_u\) equally in the two quadrature channels (real and imaginary) while all the desired signal energy \(E_b\) is collected into one of them (the real channel in our analysis since \(\mu\) is real). This processing gain advantage has not always been recognized even if complex waveforms are common for reasons other than to increase the processing gain.

V. BIT ERROR RATE PERFORMANCE

The signal-to-noise ratio result of (13) that we obtained for binary antipodal signaling suggests the bit error probability performance \(P_b = \frac{1}{2} \text{erfc} \sqrt{\text{SNR}}\). However, there is no assurance that the detection output \(z\) of (6) is a Gaussian random variable when considering arbitrary interference. Gaussian interference is not excluded so one can claim, considering the max-min performance objective, the lower bound

\[
P_b \geq \frac{1}{2} \text{erfc} \sqrt{E_b/(N_0 + \alpha E_u \Delta/T)}.
\]

The main purpose of our analysis is to determine an upper bound for max-min \(P_b\). For this task it is necessary to consider the detailed structure of the pseudo-random spreading waveform. The Chebyshev inequality in statistics would provide us with such a bound, but it is poor and can be greatly improved whenever the moment generating function \(E \exp(t \text{Re } z)\) exists to yield the Chernoff bound familiar to coding theory. In Appendix A, the Chernoff bound is derived and applied to four-phase shift keyed (QPSK) spreading waveform modulation with independent equally probable phase hops to obtain a proper waveform. The bit error probability bound

\[
P_b \leq \exp \left[-E_b/(N_0 + E_u \Delta/T)\right]
\]

is obtained where it should be noted that no restriction has been placed on the interference signal \(\sqrt{E_u} \text{Re } \hat{u}(t)\) more than it is limited to the energy \(E_u\) (or less) over symbol period \((0, T)\).

It is not difficult to perceive that, in general, for PSK spreading waveforms, the bound max-min \(P_b \leq \exp(-\text{SNR})\) applies with SNR given by (13). This upper bound is quite close to the lower bound of (14) for \(\alpha = 1\). Actually, the difference is less than 2 dB for bit error rates less than \(1 \times 10^{-3}\). See Figure 2.

With the upper bound (15) applicable for the most general interference situation, it still seems that one should be able to improve on the Chernoff bound in view of the central limit theorem if some additional constraints on the interference are imposed. A lower upper bound can be obtained if uniform interference intensity over a certain portion of the symbol period can be justified. Using the Fourier representation \(s(t) = \sum s_k \psi_k\) defined in Appendix A, we may express \(\text{Re } \Sigma u_k s_k^* = u^* s\) where \(u \) and \(s\) are two \(N\)-dimensioned vectors with the components \([\text{Re } u_1, \text{Im } u_1, \text{Re } u_2, \text{Im } u_2, \ldots]\) and \([\text{Re } s_1, \text{Im } s_1, \text{Re } s_2, \text{Im } s_2, \ldots]\). Then if the interference is uniformly distributed over \(n \ll N = 2T/\Delta\) of the dimensions,

\[
P_b < \frac{1}{2} \text{erfc} \sqrt{(n - 1)/(n - 3)} \sqrt{E_b/(N_0 + E_u (\Delta/T)n/(n - 3))}
\]
follows provided \( n \leq N = 2T/\Delta \) and \( n > 3 \). This upper bound becomes inseparable from (15) for large \( n \)-values making an analysis based on Gaussian statistics virtually correct. However, this result was achieved by the additional constraint of uniform interference over the \( n/N \)-portion of the transmitted symbol and thus lacks the general applicability of (15). In Figure 3 the closeness of (17) to the Gaussian performance graph is shown for \( n = 32 \) assuming no white noise interference.

In the derivation above the number of dimensions \( N = 2T/\Delta \) corresponds to the number of independent components of the spreading waveform over the symbol period \((0, T)\). In other words, \( N \) represents the degrees of freedom being exercised within the symbol and generally the performance of a spread spectrum communications system improves with \( N \) as well as the required transmission bandwidth. If, however, only one or a few degrees of freedom are used per symbol out of many, as in the case of frequency hopping spreading waveform modulation, the upper performance bound [3] becomes drastically different from (15) and (17).

VI. CONCLUSIONS

The best linear processing, considering the class of all interference signals with finite energy over the symbol period for spread spectrum communications, is achieved by matched signal processing. Conceptually, this result is only true having postulated the receiver processing first and thus defined its interference processing properties followed by matching the transmitted signal to the receiver correlation process. The class of all interference signals of finite energy includes all finite power signals, both random and those lacking statistical description, and this very general result was obtained by considering (pseudo-) random spreading waveforms to provide the statistical formulation of the performance analysis. To maximize the worst case performance, one should minimize the maximum characteristic value or the maximum spectral density of the spreading waveform. From a system design point of view, the min-max spreading waveform optimization is constrained by the available symbol time-bandwidth product [3, 6]. It is also noteworthy that a 3 dB performance advantage is obtained by using a proper spreading waveform with both in phase and quadrature components.

Digital spread spectrum communications performance analysis in arbitrary interference presents a distribution theory problem which by nature is intractable. Therefore, upper and lower performance bounds have been presented which for QPSK spreading waveform modulation yield bounds that are exponentially tight, within 2 dB for bit error probabilities of less than \( 1 \times 10^{-8} \). In the restricted case of uniform interference, at least in the average, interference it is shown that the two bounds approach the performance one would obtain assuming Gaussian interference.

APPENDIX A--DERIVATION OF BIT ERROR BOUND

The Chernoff bound is closely connected with the properties of the moment generating function \( g(t) = E \exp \{ tx \} \) of a random variable \( x \) defined by its probability measure \( P \). Now since \( g(t) > 0 \), then \( F(dx) = g^{-1}(t) \exp \{ tx \} P(dx) \), makes \( F \) a probability measure. This fact allows us to obtain the bound

\[
Pr \{ x < a \} = \int_0^a \exp \{ -t(x - a) \} F(dx)
\]

\[
\leq g(t) \exp \{ -at \} \quad \text{for } t \leq 0 \quad (A1)
\]

since \( \exp \{ -t(x - a) \} \leq 1 \) for \( t(x - a) \geq 0 \). In our case the bit error probability \( P_b = Pr \{ \Re(z) \leq 0 \} \) with \( z \) given by (8), assuming \( m = 1 \), is desired for which the Chernoff bound is obtained by minimizing (A1) over \( t \leq 0 \).

For QPSK spreading we introduce the set \( \{ \Psi_k(t) \} \) of orthogonal normal basis functions defined by \( \Psi_k(t) = \sqrt{1/\Delta} \) for \( k \Delta < t < (k + 1) \Delta \) and zero elsewhere, which allows us to write

\[
s(t) = \sum s_k \Psi_k(t)
\]

where \( s_k = (x_k + iy_k)\sqrt{\Delta/2T} \) and \( x_k \) and \( y_k \) are independent random variables taking the values \( \pm 1 \) with equal probability. With this representation of \( s(t) \), we can express \( z \) of (8) as \( z = \sqrt{E_b + \sum \Re u_k s_k^*} + \sqrt{\sum \Im u_k s_k^*} \) where \( \{u_k\} \) and \( \{w_k\} \) are the Fourier coefficients corresponding to \( u(t) \) and \( w(t) \) with respect to the basis functions \( \{\Psi_k(t)\} \).

(There is no need to ensure completeness of the set \( \{\Psi_k\} \).)

Now given a particular interference waveform \( u(t) \) (i.e., a set \( \{u_k\} \) the conditioned expectation \( E \{ \exp (t \Re(z)) \mid u(t) \} \),

\[
s(t) = \exp (t \sqrt{E_b + \sum u_k^2}\Delta/2T \Re \sum u_k s_k^*)
\]

which is the expectation with respect to the white noise process. Since \( \Re \sum u_k s_k = \sum \Re(u_k) x_k + \Im(u_k) y_k)\sqrt{\Delta/2T} \) it follows from the fact that \( x_k \) and \( y_k \) are independent \( \pm 1 \) random variables that \( E_0(t) = \exp (t \Re(z)) \mid u(t) \) = \( \exp (t \sqrt{E_b + \sum u_k^2}\Delta/2T \Pi \cdot \cosh (t \sqrt{E_b + \sum u_k^2}\Delta/2T \Re(u_k)) \Pi \cosh (t \sqrt{E_b + \sum u_k^2}\Delta/2T \Im(u_k)) \leq \exp (t \sqrt{E_b + \sum u_k^2}\Delta/2T \sum \Re u_k) \) since \( \cosh(x) \leq (x^2/2) \). We see that \( g_0(t) \) indeed depends on \( u(t) \) but only through Figure 3.
(\Delta/T) \Sigma |u(t)|^2 = E \Sigma u(t) \delta_{t}^* |^2 \\
= \int_{0}^{T} u(t')K_{u}(t-t')u^*(t) dt dt' \\
= \int_{-\infty}^{\infty} |U(f)|^2 S(df) \leqslant \max S'(f) = \Delta/T 
\]

where \( U(f) \) is defined by (11). Therefore, \( E_0(t) \leq g(t) = \exp[-\sqrt{E_0} + (\alpha^2/4)(N_0 + E_u \Delta/T)] \)
for all interference waveforms \( \sqrt{E_u}u(t) \) of energy \( E_u \) (or less) over the bit period. So, we obtain the bit error probability bound

\[ P_b \leq \min g(t) = \exp[-E_0/(N_0 + E_u \Delta/T)] \]

A slight improved bound, in particular for high error probabilities, can be obtained by considering \( E \cosh (\alpha x) \) for a zero mean symmetrical random variable \( x \) and get \( Pr \{ x \geq a \} \leq \min g(t)/2 \cosh (\alpha t) \) for \( a \geq 0 \).

**REFERENCES**


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**I. INTRODUCTION**

An investigation in the feasibility of using the spread spectrum pseudo-noise coding techniques as a means of providing random multiple access communications through a satellite was initiated by NASA/GSFC. An experimental program was thus formulated and consisted of a transmitting/receiving system in conjunction with a satellite and two geographically separated earth stations, one at Rosman, N.C., and one at Mojave, California. A different pseudo-noise code was assigned to each earth station (1). A number of these tests were performed to establish the feasibility of the spread spectrum technique and consisted of the following types of tests: maximum number of users to access a satellite, CNR vs. SNR for limiter suppression in a spread spectrum mode, SNR degradation vs carrier frequency separation and spread spectrum signal sharing simultaneously with a TV signal.

The basic principle used in this system in the transmission of a spread spectrum signal was the redistribution of the signal energy from a narrow bandwidth \( B_t \) to that of a much wider bandwidth \( B_s \). This process was performed by bi-phase modulating a constant envelope carrier in a double balanced modulator. This signal was then translated to the RF frequency, amplified, and transmitted in the conventional manner.

In the receiving process the spread spectrum signal was first amplified and then down converted to an IF frequency of 70 MHz. The received signal was next mixed in a balanced modulator with a replica PN coded signal and the clock was phase swept over a small range to permit the correlation of the local PN signal with that of the received PN signal. Conventional demodulation of the message information followed the correlation detection process of the spread spectrum signal.

In transmitting a spread spectrum signal, the power output is spread across a bandwidth \( B_t \) as a \( X^2/X^2 \) function. A clock frequency \( f_c \) was determined which allowed only the energy in the main lobe between the first two nulls to pass through the available system bandwidth \( B_s \) (30 MHz) for the ATS experiment. In the spreading process of the original signal the power density in the bandwidth \( B_t \) (20 kHz) is reduced to a lower density in the bandwidth \( B_s \). When the received spread spectrum signal is processed by auto-correlation the total energy in the main lobe of the sin^2 X/X^2 envelope is redistributed into the smaller bandwidth \( B_t \). The ratio of the power density in the correlated bandwidth \( B_t \) to the power density at the center of the RF bandwidth \( B_s \) defines the processing gain \( R \). (2) (3) Experimental results also show that the performance characteristics conform closely to the values obtained from expressions that use a processing gain of \( B_s/B_t \). This ratio of \( R \) was thus used in the computations for these experiments. In all of the experiments a PN coded sequence was utilized having a length \( L \) of 2047 bits \( (2^n - 1) \) which was provided by an 11 stage modulo 2 shift register. (4) When a number of users are accessing a satellite simultaneously the resulting CNR for limiting conditions is expressed as (5)

\[ \text{CNR} = \frac{\alpha_0 LR}{K_c(M - 1)(L + R)} \]

The actual equipment designed for these experiments degrades the CNR by a fixed correlation loss of \(-2.8 \text{ dB} \). This...